Prof. Dr. Alfred Toth

Semiotic Ghost Trains

Ich bin hier, mehr weiss ich nicht, mehr kann ich nicht tun. Mein Kahn ist ohne Steuer, er fährt mit dem Wind, der in den untersten Regionen des Todes bläst.

(I am here, more I don't know, more I can't do. My boat is without steer, it drives with the wind, which blows in the lowest regions of death.)

Franz Kafka, Der Jäger Gracchus (1985, S. 288)

This book is dedicated to the memory of Philippe Steiner (1952-2007).

Contents

Fo	preword	7
1.	A new formal model of theoretical semiotics	9
	1.1. How many representation schemes are there?	9
	1.2. Sign classes, reality thematics and transpositions	13
	1.3. Static and dynamic categories	14
	1.4. Transitional classes	17
	1.5. Complex sign classes	21
2.	Sign connections	28
	2.1. Static sign connections	28
	2.1.1. Intra-semiotic connections	28
	2.1.2. Trans-semiotic connections	28
	2.2. Dynamic sign connections	29
	2.2.1. Intra-semiotic connections	29
	2.2.2. Combinations of transpositions and dual transpositions	31
3.	Semiotic ghost trains	41
	3.1. Transpositional realities	41
	3.2. The semiotic ghosts	44
	3.3. The semiotic rails	54
	3.3.1. Semiotic bigraphs	54
	3.3.2. Parallel tracks	60
	3.3.3. Joints and crossings	64
	3.3.4. Detours	67
	3.3.5. Returns	68
	3.3.6. Stub tracks	70
Bil	bliography	73

Foreword

This book introduces a new model of theoretical semiotics based on mathematical category theory. It is shown that each of the 10 sign classes of Peirce-Bensean semiotics has 6 transpositions, which can occur in the 4 quadrants of the Gaussian number field. Since sign classes can be built by combining different types of positive and negative sub-signs, we get a representation structure of 276 sign classes and reality thematics for each one of the original 10 sign classes and reality thematics, thus totally 552 semiotic schemes. Moreover, dynamic categorial analysis of sign sets is introduced, which allows together with static categorial analysis to handle both semiotic systems and processes in a hitherto unknown operative manner.

While the fundamentals of mathematical semiotics are presented in Chapter 1, Chapter 2 throws a special focus on monadic, dyadic and triadic sign connections which have never been studied in a systematical manner before. Chapter 3 presents the formal basis of "semiotic ghost trains"¹ representing very complex semiotic networks of "trucks" as paths and of "ghosts images" as nodes, which are identified with the structural realities presented in the reality thematics of transpositional sign classes and the different kinds of connections and transitions between them.

My heartfelt thanks go to Professor Dr. Ernst Kotzmann and Amtsrätin Andrea Lassnig (University of Klagenfurt) for the good job that they did again to turn my manuscript into a book and give it a home.

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¹ This notion is taken from two former publications by the present author: "Die Wiener Prater-Geisterbahn zu Basel" (together with H.H. Hoppel and Philippe Steiner; Zurich 1998), and "Geisterbahnsemiotik", in: Semiotische Berichte 24, 2000, pp. 381-402.

1. A new formal model of theoretical semiotics

1.1. How many representation schemes are there?

According to Peirce, a **sign** (SR) is a triadic relation over a monadic, a dyadic and a triadic relation, i.e. a relation over three relations:

 $SR = (((.1.) \Longrightarrow (.1. \Longrightarrow .2.)) \Longrightarrow (.1. \Longrightarrow .2. \Longrightarrow .3.))$

A **sub-sign** is obtained by mapping the three sign relations (.1., .2., .3.) into themselves. The Cartesian procucts are displayed in the following semiotic matrix:

	.1	.2	.3
1.	1.1	1.2	1.3
2.	2.1	2.2	2.3
3.	3.1	3.2	3.3

The rows are called **triadic values** and the columns **trichotomic values** of the matrix. In order to build a **sign class**, one sub-sign has to be taken out of each of the three rows, the rows thus being different. Therefore, **sign sets** like

* (3.1 3.2 1.3) * (2.1 2.2 1.2) * (1.1 1.3 3.1)

are not considered sign classes. Generally, sign sets must fulfill the following three requirements in order to form sign classes:

- 1. Principle of Triadic Diversity: The general sign class structure has the form (a.b c.d e.f) with a, b, c, d, e, $f \in \{1, 2, 3\}$ and $a \neq b \neq c$.
- 2. **Principle of Degenerative Triadic Order**: The general sign class structure must have the form (3.a 2.b 1.c).
- 3. Principle of Trichotomic Inclusion: (3.a 2.b 1.c) with $a \le b \le c$ and $a, b, c \in \{.1, .2, .3\}$.

Without these three restrictions we would have for the general sign class structure the 3 positions (a.b), (c.d), (e.f) each of which could be assigned by all 9 sub-signs from the semiotic matrix, hence $3^9 = 19'683$ sign classes. If we apply the Principle of Triadic Diversity, the combinations are drastically reduced to $3^3 = 27$ sign classes, since now only 3 sub-signs can be assigned to each of the 3 positions:

3.1 2.1 1.1	3.2 2.1 1.1	3.3 2.1 1.1
3.1 2.1 1.2	3.2 2.1 1.2	3.3 2.1 1.2
3.1 2.1 1.3	3.2 2.1 1.3	3.3 2.1 1.3
3.1 2.2 1.1	3.2 2.2 1.1	3.3 2.2 1.1
3.1 2.2 1.2	3.2 2.2 1.2	3.3 2.2 1.2
3.1 2.2 1.3	3.2 2.2 1.3	3.3 2.2 1.3
3.1 2.3 1.1	3.2 2.3 1.1	3.3 2.3 1.1
3.1 2.3 1.2	3.2 2.3 1.2	3.3 2.3 1.2
3.1 2.3 1.3	3.2 2.3 1.3	3.3 2.3 1.3

If we also apply the Principle of Trichotomic Inclusion, these 27 combinations are reduced to the following 10 sign classes that build the core system of Peirce-Bensean semiotics:

3.1 2.1 1.1	3.1 2.3 1.3
3.1 2.1 1.2	3.2 2.2 1.2
3.1 2.1 1.3	3.2 2.2 1.3
3.1 2.2 1.2	3.2 2.3 1.3
3.1 2.2 1.3	3.3 2.3 1.3

Furthermore, each sign class can be assigned its dual "sign class", called **reality thematic**. The semiotic operation of **dualization** ("×") inverts both the triadic and the trichotomic order:

 \times (a.b c.d e.f) = (f.e d.c b.a)

$(3.1\ 2.1\ 1.1) \times (1.1\ 1.2\ 1.3)$	$(3.1\ 2.3\ 1.3) \times (3.1\ 3.2\ 1.3)$
$(3.1\ 2.1\ 1.2) \times (2.1\ 1.2\ 1.3)$	$(3.2\ 2.2\ 1.2) \times (2.1\ 2.2\ 2.3)$
$(3.1\ 2.1\ 1.3) \times (3.1\ 1.2\ 1.3)$	$(3.2\ 2.2\ 1.3) \times (3.1\ 2.2\ 2.3)$
$(3.1\ 2.2\ 1.2) \times (2.1\ 2.2\ 1.3)$	$(3.2\ 2.3\ 1.3) \times (3.1\ 3.2\ 2.3)$
$(3.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.3)$	$(3.3\ 2.3\ 1.3) \times (3.1\ 3.2\ 3.3)$

However, we may question if the three semiotic restrictions do hold. From the standpoint of relational algebra, nothing speaks in favor of the Principle of Semiotic Inclusion. In the opposite, the semiotic matrix contains as its main diagonal the sign set (3.3 2.2 1.1), which is not built according to this principle and was called by Bense "the normed leading semiosis of all sign processes in general" (1975, p. 89). On the level of dyadic sub-sign sets, too, there is no restriction that forbids non-inclusive combinations (Bense 1975, pp. 100 ss.):

(1.1 2.1), (1.1 2.2), (1.1 2.3); (2.1 3.1), (2.1 3.2), (2.1 3.2), etc.

In the "Large Semiotic Matrix" (Bense 1975, p. 105), all possible dyadic sign combinations show up:

(1.1 1.1), (1.1 1.2), (1.1 1.3); (1.1 2.1), (1.1 2.2), (1.1 2.3); (1.1 3.1), (1.1 3.2), (1.1 3.3) (1.2 1.1), (1.2 1.2), (1.2 1.3); (1.2 2.1), (1.2 2.2), (1.2 2.3); ... (1.3 1.1), (1.3 1.2), (1.3 1.3);

(3.3 1.1), (3.3 1.2), ...

A bigger problem is exhibited by the Principle of Triadic Diversity, which guarantees that each sign has an object to which it refers and an interpretant, which forms the context. However, as Nöth (1980, pp. 72 ss.) pointed out, there are signs without object, without interpretant and even without medium (1980, p. 89). Therefore, it makes perfect sense to consider sign sets like (1.2 1.3 1.1), (2.2 2.1 1.3) or (3.2 3.3 2.1) sign classes – the more as there are no mathematical and logical obstacles against an abolishment of this second semiotic restriction, either.

The Principle of Degenerative Triadic Order, which is based on the Principle of Triadic Diversity, was abolished by Bense himself (1971, pp. 37 ss.). We will dedicate Chapter 1.2 to this problem.

In the following, we shall show that the three systems of sign classes – the one with 10, the one with 27 and the one with 19'683 sign classes – is strongly connected to Günther's classification of polycontextural numbers into the three systems of proto-, deutero- and trito-numbers (Günther 1979, pp. 241 ss.). Peirce-Bensean semiotics, therefore, is relevant to Polycontextural Theory, an idea that was already brought up by Maser (1973, pp. 29 ss.), but denied by Bense (1975, p. 22).

"Proto-Structure emerges from the requirement to build up the vertical series of kenograms under the viewpoint that only an absolute minimum of repetition occurs in their structure (...). We further stipulate that the placing of individual kenograms may be arbitrary in a given vertical series" (Günther 1980, p. 111).

"Deutero-Structure results from the assumption that maximal repetition is allowed for individual kenograms. As for the rest, the placing of the symbol still remains irrelevant" (Günther 1980, p. 111).

"Trito-Structure differs from proto- and deutero-structure because the position of a symbol in the vertical sequence gets relevant. As for the rest, here, too, maximal repetition of a given symbol is allowed (...). By the relevance of the position of a symbol the trito-structure differs fundamentally from the two preceding structures" (Günther 1980, p. 112).

Let's now have a look at our three systems of sign classes. In the system of 19'683 sign classes, because of the free combination of all sub-signs from the semiotic matrix, we will not only find a given structure (a.b c.d e.f), but also all of its transpositions, hence (a.b e.f c.d), (c.d a.b e.f), (c.d e.f a.b), (e.f a.b c.d) and (e.f c.d a.b) which are semiotically not identical (cf. Chapter 1.2), i.e. the position of the sub-sign in each of the 19'683 sign classes is

relevant. Since all sub-sign combinations show up, maximal repetition is granted.² Therefore, the system of the 19'683 sign classes fulfills the requirements of trito-numbers.

As for the 27 sign classes, the Principle of Triadic Diversity prevents maximal repetition of the sub-signs, and the Principle of Degenerative Triadic Order which excludes generative and mixed orders of the sequences of the sub-signs leads to the conclusion that here the placing of a sub-signs is irrelevant, since transpositional sequences like (2.3 3.2 1.2) will be brought automatically into the degenerative order (3.2 2.3 1.2). Therefore, the system of the 27 sign classes takes an intermediary position between the system of deutero- and tritonumbers.

Since the Principle of Trichotomic Inclusion not only limits the repetition of the sub-signs, but also requires that only a limited subset of the sub-signs are combined with one another, the system of the 10 sign classes is closer to proto- than to trito-numbers, although here, too, the placing of the sub-signs is free insofar as deviant sequences are automatically brought into the "right" order by the Principle of Degenerative Triadic Order. We therefore conclude that the three semiotic systems are all polycontextural insofar as the order of the sub-signs of their sign classes is irrelevant, but monocontextural in the systems of the 27 and 10 sign classes can be considered fully polycontextural while the systems of the 27 and 10 sign-classes take an intermediary position between poly- and monocontextural number systems.

This intermediary position of semiotics between poly- and monocontexturality does not only show up in the different systems of sign classes, but already in the semiotic matrix given above. When Bense states that "the semiotic matrix fixes the phases of the flow of abstraction between reality and conscience as phases of semioses with the stable moments of abstraction as signs, i.e. as modified states of reality in the sense of modified states of conscience" (1975, p. 39), then the signs are settled in the intermediary region between world and conscience and "even that interface between "praesentamen" and "repraesentamen" is taken into the thesis of signs by the sign" (Bense 1979, p. 19). Hence the signs settle the Hegelian space of Becoming (Werden) between Being (Sein) and Nothing (Nichts) where we find a network of mono- and polycontextural structures sketched above.

Let us therefore have a closer look at the semiotic matrix:

² As a matter of fact, the system of the 19'683 sign classes contains all transpositions of all sign classes. Since homogeneous sign classes like (1.1 1.1 1.1) are identical with all of their transpositions and since there are 9 such homogeous sign classes, we get 19'674 inhomogeneous sign classes. Since each sign class has 6 transpositions, we get 3'279 "basic" sign classes.



The respective successors of the sub-signs are therefore:

$$\begin{split} & S(1.1) = (1.2), (2.1), (2.2) \\ & S(1.2) = (1.3); (2.1), (2.2), (2.3) \\ & S(1.3) = (2.2), (2.3) \\ & S(2.1) = (2.2); (3.1), (3.2) \\ & S(2.2) = (2.3); (3.1), (3.2), (3.3) \\ & S(2.3) = (3.2), (3.3) \\ & S(3.1) = (3.2) \\ & S(3.2) = (3.3) \end{split}$$

Apparently, there are two kinds of successors of these semiotic numbers: successors of the same and of different triadic value. Following Günther's terminology about polycontextural numbers (1979, p. 280 ss.), we will call them **intra- and transcontextural semiotic successors**. Furthermore, for a semiotic number we have maximally the following types of successors:

$$S(a.b) = \begin{cases} (a+1.b), \text{ f.ex. } S(2.2) = (3.2) \\ (a.b+1), \text{ f.ex. } S(2.2) = (2.3) \\ (a+1.b-1), \text{ f.ex. } S(2.2) = (3.1) \\ (a+a+1.b+1), \text{ f.ex. } S(2.2) = (3.3) \end{cases}$$

Thus, semiotic numbers have 0, 1, 2, 3 or 4 successors and therefore do not follow the linearity of the Peano numbers; we will call them **Peirce numbers**.

1.2. Sign classes, reality thematics and transpositions

The Principle of Degenerative Triadic Order states that each sign class must have the form

but if we have a look at the reality thematics:

(c.1 b.2 a.3),

where a, b, $c \in \{1., 2., 3.\}$, we get sign sets which contradict both the Principle of Degenerative Triadic Order and the Principle of Triadic Diversity:

(1.1 1.2 1.3) × (3.1 2.1 1.1) (2.1 1.2 1.3) × (3.1 2.1 1.2) (3.1 1.2 1.3) × (3.1 2.1 1.3), etc.

The only exception amongst the system of 10 sign classes is the dual-invariant sign class

 $(3.1\ 2.2\ 1.3) \times (3.1\ 2.2\ 1.3).$

So, besides this one exception, either reality thematics must not be considered sign classes or the Principles of Degenerative Triadic Order and of Triadic Diversity must be abolished. Since Bense (1971, pp. 40, 102; 1976, p. 107) considers

- the "generative" graph $(1.c \rightarrow 2.b \rightarrow 3.a)$
- the "thetic" graph $(3.a \rightarrow 1.c \rightarrow 2.b)$
- the "communicative" graph $(2.b \rightarrow 1.c \rightarrow 3.a)$
- the two "creative" graphs $(3.a \rightarrow 1.c \rightarrow 2.b)$ and $(1.c \rightarrow 3.a \rightarrow 2.b)$

sign classes, the Principle of Degenerative Triadic Order is abolished; from all possible transpositions of the general sign structure (3.a 2.b 1.c) only the following typ of semiotic ordering has to be defined

- $(2.b \rightarrow 3.a \rightarrow 1.c)$

It follows that all transpositions of a sign class are sign classes, too, and of course, since reality thematics follow the order of generative graphs, reality thematic are sign classes, too. Each sign class has therefore 6 transpositions and 6 reality thematics. If we also consider that reality thematics do not fulfill the Principle of Triadic Diversity, we get the following system for each general sign structure (a.b c.d e.f) with a, ..., $f \in \{1, 2, 3\}$:

 $(a.b c.d e.f) \times (f.e d.c b.a)$ $(a.b e.f c.d) \times (d.c f.e b.a)$ $(c.d a.b e.f) \times (f.e b.a d.c)$ $(c.d e.f a.b) \times (b.a f.e d.c)$ $(e.f a.b c.d) \times (d.c b.a f.e)$ $(e.f c.d a.b) \times (b.a d.c f.e)$

1.3. Static and dynamic semiotic categories

Following Bense (1981, pp. 124 ss.), we may assign a semiotic category to each one of the sub-signs considering that there is a dual category to each subsign but the identitive ones and that two categories together with their inverses and the categorial composition laws are sufficient:

	.1	.2	.3
1.	id1	α	βα
2.	α°	id2	β
3.	α°β°	β°	id3

Therefore, each sign class is assigned its categorial structure by simple replacement of the constitutive sub-signs by their respective categories, f.ex.

 $(3.1\ 2.1\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$

But let us have a look at the categorial schemes of the system of the 10 sign classes:

 $\begin{array}{l} (3.1\ 2.1\ 1.1) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \mathrm{id}1] \\ (3.1\ 2.1\ 1.2) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \alpha] \\ (3.1\ 2.1\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha] \\ (3.1\ 2.2\ 1.2) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \mathrm{id}2, \alpha] \\ (3.1\ 2.2\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \mathrm{id}2, \beta\alpha] \\ (3.1\ 2.3\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \beta, \beta\alpha] \\ (3.2\ 2.2\ 1.2) \Leftrightarrow [\beta^{\circ}, \mathrm{id}2, \alpha] \\ (3.2\ 2.3\ 1.3) \Leftrightarrow [\beta^{\circ}, \beta, \beta\alpha] \\ (3.3\ 2.3\ 1.3) \Leftrightarrow [\beta^{\circ}, \beta, \beta\alpha] \\ (3.3\ 2.3\ 1.3) \Leftrightarrow [\mathrm{id}3, \beta, \beta\alpha] \end{array}$

The categorial structures of the sign classes do not tell as anything about the three principles of sign classes. Neither Triadic Diversity, nor Degenerative Triadic Order, nor Trichotomic Inclusion is visible in the categorial notations. But most of all, the ascription of categories to sub-signs is **static**, not taking into consideration the definition of the sign as a triadic relation over a monadic, a dyadic and a triadic relation.

In order to cope with the fact that a sign is a relation over relations, we introduce dynamic semiotic categories. Since each triadic relation

(a.b c.d e.f)

can be noted as an intersection of two dyadic relations (cf. Walther 1979, p. 79):

 $(a.b c.d) \cap (c.d e.f),$

we have to ascribe categories to the semiotic transitions between the two triadic relations

(a. \rightarrow c.), (c. \rightarrow e.)

and of the two trichotomic relations

 $(.b \rightarrow .d), (.d \rightarrow .f),$

which establish together the complete abstract sign structure by an operation called "associative addition" by Bense (1981, p. 204). The categories ascribed to the triadic and trichotomic transitions inside of a sign-class (or sign set) are **dynamic** since they take into account the processual nature of sign classes with their constitutive sub-signs as static moments. We therefore get the following dynamic categorial structures for the 10 sign classes:

 $\begin{array}{l} (3.1 \ 2.1 \ 1.1) \Leftrightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \\ (3.1 \ 2.1 \ 1.2) \Leftrightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \\ (3.1 \ 2.1 \ 1.3) \Leftrightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \\ (3.1 \ 2.2 \ 1.2) \Leftrightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \\ (3.1 \ 2.2 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \\ (3.1 \ 2.3 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]] \\ (3.2 \ 2.2 \ 1.2) \Leftrightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]] \\ (3.2 \ 2.3 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]] \\ (3.2 \ 2.3 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]] \\ (3.3 \ 2.3 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]] \end{array}$

The dynamic categorial analysis of the system of the 10 sign classes, unlike the static one, exhibits clearly

- the Principle of Triadic Diversity by the general categorial structure [[β° , X], [α° , Y]
- the Principle of Degenerative Triadic Order by $[[\beta^{\circ}, X], [\alpha^{\circ}, Y]]$ with $[\beta^{\circ}] > [\alpha^{\circ}]$
- the Principle of Trichotomic Inclusion by the fact that in the structure $[[\beta^{\circ}, X], [\alpha^{\circ}, Y]$ the categories symbolized by X and Y are never inverse

To conclude this chapter, let us have a look at the system of 6 transpositions and their 6 reality thematics won by transposition in Chapter 1.2. As an example, we take the sign class (3.1 2.1 1.3):

$(3.1\ 2.1\ 1.3) \times (3.1\ 1.2\ 1.3)$	\Leftrightarrow	$[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \times [[\alpha^{\circ}\beta^{\circ}, \alpha], [id1, \beta]]$
$(3.1\ 1.3\ 2.1) \times (1.2\ 3.1\ 1.3)$	\Leftrightarrow	$[[\alpha^{\circ}\beta^{\circ},\beta\alpha],[\alpha,\alpha^{\circ}\beta^{\circ}]]\times[[\beta\alpha,\alpha^{\circ}],[\alpha^{\circ}\beta^{\circ},\beta\alpha]]$
$(2.1 \ 3.1 \ 1.3) \times (3.1 \ 1.3 \ 1.2)$	\Leftrightarrow	$[[\beta, \mathrm{id}1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] \times [[\alpha^{\circ}\beta^{\circ}, \beta\alpha], [\mathrm{id}1, \beta^{\circ}]]$
$(2.1\ 1.3\ 3.1) \times (1.3\ 3.1\ 1.2)$	\Leftrightarrow	$[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \times [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\alpha^{\circ}\beta^{\circ}, \alpha]]$
$(1.3\ 3.1\ 2.1) \times (1.2\ 1.3\ 3.1)$	\Leftrightarrow	$[[\beta \alpha, \alpha^{\circ} \beta^{\circ}], [\beta^{\circ}, id1]] \times [[id1, \beta], [\beta \alpha, \alpha^{\circ} \beta^{\circ}]]$
$(1.3\ 2.1\ 3.1) \times (1.3\ 1.2\ 3.1)$	\Leftrightarrow	$[[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]] \times [[id1, \beta^{\circ}], [\beta\alpha, \alpha^{\circ}]]$

Generally, we see that reality thematics of each transposition are won by reversing the order of the whole natural transformation and of its categories which are also reversed, i.e. $X \rightarrow X^{\circ}, X^{\circ} \rightarrow X$.

1.4. Transitional classes

Whatever categorial structure we ascribe to a sign class like (3.1 2.1 1.3), the static one:

$$(3.1\ 2.1\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$$

or the dynamic one:

 $(3.1 \ 2.1 \ 1.3) \Leftrightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]],$

one must be aware that sign classes never occur alone, but are always part of a sign system, if they are connected to one another by common sub-signs or not. Since it is clear that connections between sign classes can only be sign classes or sign sets, we must find a way to determine the intermediary sign sets that form the transitions between sign classes or sign sets. Consider the three possibilities to note sign classes: the numerical and the two categorial ones:

(3.1 2.1 1.3)	$[\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$	[[β°, id1], [α°, βα]]
(3.2 2.2 1.2)	[β°, id2, α]	[[β°, id2], [α°, id2]]

On numerical and static-categorial level there are no connections visible between the sign classes (3.1 2.1 1.3) and (3.2 2.2 1.2), but the dynamic-categorial structure shows the functor $[\beta^{\circ}, \alpha^{\circ}]$ whose numerical value is (3.2, 2.1). Therefore, $[\beta^{\circ}, \alpha^{\circ}]$ or (3.2, 2.1) can be interpreted as transitional (sign) class between the two sign classes.

In the system of the 10 sign classes there are exactly 45 transitional classes. Since they can be reduced to fewer types recurring more than once between different sign classes, we list them here completely:

 $(3.1 \ 2.1 \ 1.1) \rightarrow (3.1 \ 2.1 \ 1.2) \iff [[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]]$ Transitional class: $[\beta^{\circ}, id1, \alpha^{\circ}] \Leftrightarrow (3.2 \ 1.1 \ 2.1)$

 $\begin{array}{ll} (3.1 \ 2.1 \ 1.1) \rightarrow (3.1 \ 2.1 \ 1.3) & \Leftrightarrow & [[\beta^{\circ}, \operatorname{id}1], [\alpha^{\circ}, \operatorname{id}1]] \rightarrow [[\beta^{\circ}, \operatorname{id}1], [\alpha^{\circ}, \beta\alpha]] \\ \text{Transitional class:} & [\beta^{\circ}, \operatorname{id}1, \alpha^{\circ}] \Leftrightarrow (3.2 \ 1.1 \ 2.1) \end{array}$

 $\begin{array}{ll} (3.1 \ 2.1 \ 1.1) \rightarrow (3.1 \ 2.2 \ 1.2) & \Leftrightarrow & [[\beta^{\circ}, \operatorname{id}1], [\alpha^{\circ}, \operatorname{id}1]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \operatorname{id}2]] \\ \text{Transitional class:} & [\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1) \end{array}$

 $\begin{array}{l} (3.1 \ 2.1 \ 1.1) \rightarrow (3.1 \ 2.2 \ 1.3) & \Leftrightarrow \qquad [[\beta^{\circ}, \operatorname{id}1], [\alpha^{\circ}, \operatorname{id}1]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \\ \text{Transitional class: } [\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1) \end{array}$

 $(3.1\ 2.1\ 1.1) \rightarrow (3.1\ 2.3\ 1.3) \iff [[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2\ 2.1)$

 $[[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ $(3.1\ 2.1\ 1.1) \rightarrow (3.2\ 2.2\ 1.2) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ $(3.1\ 2.1\ 1.1) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ $(3.1\ 2.1\ 1.1) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.1) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, id1]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]]$ $(3.1\ 2.1\ 1.2) \rightarrow (3.1\ 2.1\ 1.3)$ \Leftrightarrow Transitional class: $[\beta^{\circ}, id1, \alpha^{\circ}] \Leftrightarrow (3.2 \ 1.1 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.1\ 2.2\ 1.2)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]]$ \Leftrightarrow Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.1\ 2.2\ 1.3)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]]$ \Leftrightarrow Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.1\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.2\ 2.2\ 1.2) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.2) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \alpha]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ $(3.1\ 2.1\ 1.2) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.3) \rightarrow (3.1\ 2.2\ 1.2) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]]$ $(3.1\ 2.1\ 1.3) \rightarrow (3.1\ 2.2\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$

 $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]]$ $(3.1\ 2.1\ 1.3) \rightarrow (3.1\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.3) \rightarrow (3.2\ 2.2\ 1.2)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ \Leftrightarrow Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.1\ 1.3) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, \beta]], [\alpha^{\circ}, id3]]$ $(3.1\ 2.1\ 1.3) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ $(3.1\ 2.1\ 1.3) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.2\ 1.2) \rightarrow (3.1\ 2.2\ 1.3) \quad \Leftrightarrow \quad [[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]]$ Transitional class: $[\beta^{\circ}, \alpha, \alpha^{\circ}] \Leftrightarrow (3.2 \ 1.2 \ 2.1)$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]]$ $(3.1\ 2.2\ 1.2) \rightarrow (3.1\ 2.3\ 1.3) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ $(3.1\ 2.2\ 1.2) \rightarrow (3.2\ 2.2\ 1.2) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}, id2] \Leftrightarrow (3.2 \ 2.1 \ 2.2)$ $(3.1\ 2.2\ 1.2) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.2\ 1.2) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, \beta]], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ $(3.1\ 2.2\ 1.2) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]]$ $(3.1\ 2.2\ 1.3) \rightarrow (3.1\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.2\ 1.3) \rightarrow (3.2\ 2.2\ 1.2)$ \Leftrightarrow $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ $(3.1\ 2.2\ 1.3) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow \quad$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}, \beta] \Leftrightarrow (3.2 \ 2.1 \ 2.3)$ $(3.1\ 2.2\ 1.3) \rightarrow (3.2\ 2.3\ 1.3) \iff [[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$

 $[[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ $(3.1\ 2.2\ 1.3) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.3\ 1.3) \rightarrow (3.2\ 2.2\ 1.2) \quad \Leftrightarrow \qquad [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, id2]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.3\ 1.3) \rightarrow (3.2\ 2.2\ 1.3) \quad \Leftrightarrow \qquad [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, \mathrm{id}3]] \rightarrow [[\beta^{\circ}, \mathrm{id}2], [\alpha^{\circ}, \beta]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.1\ 2.3\ 1.3) \rightarrow (3.2\ 2.3\ 1.3) \iff [[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}, id3] \Leftrightarrow (3.2\ 2.1\ 3.3)$ $[[\beta^{\circ}, \beta\alpha], [\alpha^{\circ}, id3]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ $(3.1\ 2.3\ 1.3) \to (3.3\ 2.3\ 1.3)$ \Leftrightarrow Transitional class: $[\beta^{\circ}, \alpha^{\circ}, id3] \Leftrightarrow (3.2 \ 2.1 \ 3.3)$ $[[\beta^{\circ}, id2], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]]$ $(3.2\ 2.2\ 1.2) \rightarrow (3.2\ 2.2\ 1.3)$ \Leftrightarrow Transitional class: $[\beta^{\circ}, id2, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.2 \ 2.1)$ $(3.2\ 2.2\ 1.2) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, id2], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow \equiv (3.2 \ 2.1)$ $(3.2\ 2.2\ 1.2) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow$ $[[\beta^{\circ}, id2], [\alpha^{\circ}, id2]] \rightarrow [[\beta^{\circ}, id3], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.2\ 2.2\ 1.3) \rightarrow (3.2\ 2.3\ 1.3) \quad \Leftrightarrow \quad$ $[[\beta^{\circ}, id2], [\alpha^{\circ}, \beta]] \rightarrow [[\beta^{\circ}, \beta], [\alpha^{\circ}, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.2\ 2.2\ 1.3) \rightarrow (3.3\ 2.3\ 1.3) \iff [[\beta^\circ, id2], [\alpha^\circ, \beta]] \rightarrow [[\beta^\circ, id3], [\alpha^\circ, id3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}] \Leftrightarrow (3.2 \ 2.1)$ $(3.2\ 2.3\ 1.3) \rightarrow (3.3\ 2.3\ 1.3) \quad \Leftrightarrow \qquad [[\beta^{\circ}, \beta], [\alpha^{\circ}, \mathrm{id}3]] \rightarrow [[\beta^{\circ}, \mathrm{id}3], [\alpha^{\circ}, \mathrm{id}3]]$ Transitional class: $[\beta^{\circ}, \alpha^{\circ}, id3] \Leftrightarrow (3.2 \ 2.1 \ 3.3)$ All possible transitions between the 10 sign classes are done by one of the following 7 transitional classes:

Dyadic transitional class:(3.2 2.1)Triadic transitional classes:(3.2 1.1 2.1), (3.2 2.1 2.1), (3.2 2.1 2.2), (3.2 2.1 2.3), (3.2 2.1 3.3), (3.2 2.2 2.1)

It is interesting to observe that among the triadic transitional classes there are solely sign classes (or sign sets) that are not built according to the three semiotic restrictions elaborated in Chapter 1.1. Thus they occur "naturally" between the regularly built sign classes as the Genuine Category Class (3.3 2.2 1.1) appears "naturally" as main diagonal of the semiotic

matrix together with the main sign classes (with identical trichotomic values) in the rows and the dual-identical sign class as secondary diagonal.

1.5. Complex sign classes

A regular sign class whose general structure is (a.b c.d e.f) has all its values in the first quadrant of a Cartesian coordinate system. From the mathematical standpoint, however, nothing impedes us mapping (a.b c.d e.f) also to the second, third and fourth quadrant, thus getting sign classes in all positive and negative parts of the Gaussian number field. Therefore, we can put the above sign structure in a maximally general form:

 $(\pm a.\pm b \pm c.\pm d \pm e.\pm f) \times (\pm f.\pm e \pm d.\pm c \pm b.\pm a)$

Since we have shown in Chapter 1.2 that there are 6 transpositions for each sign class and for each reality thematic, thus totally 12 representation schemes, mapping them onto the Gaussian number field yields 48 basic representation systems for each sign class. We will call the sign classes without negative algebraic sign **real sign classes** and the ones with negative algebraic sign **complex sign classes**:

$(a.b c.d e.f) \times (f.e d.c b.a)$	$(-a.b - c.d - e.f) \times (fe dc ba)$
$(a.b e.f c.d) \times (d.c f.e b.a)$	$(-a.b - e.f - c.d) \times (dc fe ba)$
$(c.d a.b e.f) \times (f.e b.a d.c)$	$(-c.d -a.b -e.f) \times (fe ba dc)$
$(c.d e.f a.b) \times (b.a f.e d.c)$	$(-c.d - e.f - a.b) \times (ba fe dc)$
$(e.f a.b c.d) \times (d.c b.a f.e)$	$(-e.f -a.b -c.d) \times (dc ba fe)$
$(e.f c.d a.b) \times (b.a d.c f.e)$	$(-e.f - c.d - a.b) \times (ba dc fe)$
(ab cd ef) × (-f.e -d.c -b.a)	$(-ab - cd - ef) \times (-fe - dc - ba)$
$(ab ef cd) \times (-d.c -f.e -b.a)$	$(-ab - ef - cd) \times (-dc - fe - ba)$
$(cd ab ef) \times (-f.e -b.a -d.c)$	$(-cd -ab -ef) \times (-fe -ba -dc)$
$(cd ef ab) \times (-b.a - f.e - d.c)$	$(-cd - ef - ab) \times (-ba - fe - dc)$
$(ef ab cd) \times (-d.c -b.a -f.e)$	$(-ef -ab -cd) \times (-dc -ba -fe)$
$(ef cd ab) \times (-b.a -d.c -f.e)$	$(-ef -cd -ab) \times (-ba -dc -fe)$

We notice that the reality thematics of complex sign classes from the second quadrant lie in the third quadrant and vice versa, thus exhibiting **categorial merging**; cf. f.ex.

 $\begin{array}{l} (-3.1 \ -2.1 \ -1.3) \times (3.-1 \ 1.-2 \ 1.-3) \\ (3.-1 \ 2.-1 \ 1.-3) \times (-3.1 \ -1.2 \ -1.3) \end{array}$

Furthermore, we may construct sign classes with mixed positive and negative values. Each of the three sub-signs of a sign class can have either a negative triadic and/or a negative trichotomic value:

The following 18 sign classes lie in two quadrants:

1.3
1.3
1.3
-1.3
13
-13
-

24 sign classes lie in three quadrants (the maximum for a triadic sign class):

3.1 -2.1 -13	-31 -2.1 1.3	3.1 -2.1 13
3.1 -21 -1.3	-3.1 2.1 -13	3.1 21 -1.3
-3.1 -21 1.3	-31 2.1 -1.3	-3.1 21 1.3
31 -2.1 1.3	-3.1 -21 13	31 -21 -1.3
-3.1 2.1 13	-3.1 21 -13	-31 -2.1 13
31 2.1 -1.3	-31 21 -1.3	31 -2.1 -13
3.1 -21 13 3.1 21 -13 -31 21 1.3	31 -21 1.3 -31 2.1 13 31 2.1 -13	

Thus, together with the 4 sign classes that lie in one quadrant, we get the total of 4 + 18 + 24 = 46 complex sign classes each of which has of course 6 transpositions, hence 276 sign classes, and since each sign class has its reality thematics, we have now established a structural semiotic wealth of **552 representation schemes** for each of the original 10 sign classes, thus in toto a formal semiotic model which contains 5'520 sign classes and reality thematics still obeying the Principles of Triadic Diversity and of Trichotomic Inclusion.

However, considering complex sign classes, the semiotic category theory presented in chapter 1.3 has to be enlarged, since up to now there are no morphisms that map real to complex sub-signs. Since each sub-sign has three possibilities to be negative, i.e. (-a.b), (a.-b) and (-a.-b), we introduce the apostrophe to mark mappings from reel into complex sub-signs, whereby one apostrophe marks triadically, two apostrophes trichotomically and three apostrophes both triadically and trichotomically negative sub-signs:

 $\begin{array}{l} (-1.1) \Leftrightarrow \operatorname{id1'}; (1.-1) \Leftrightarrow \operatorname{id1''}; (-1.-1) \Leftrightarrow \operatorname{id1'''} \\ (-1.2) \Leftrightarrow \alpha'; (1.-2) \Leftrightarrow \alpha''; (-1.-2) \Leftrightarrow \alpha''' \\ \cdots \\ (-3.1) \Leftrightarrow \alpha^{\circ}\beta^{\circ'}; (3.-3) \Leftrightarrow \alpha^{\circ}\beta^{\circ''}; (-3.-3) \Leftrightarrow \alpha^{\circ}\beta^{\circ'''} \\ \cdots \end{array}$

This simple notation allows us to turn a complex sign class like, f.ex., (3.-1 - 2.-1 1.3) into its corresponding categorial form:

 $[[\beta^{\circ}, id1^{\circ}], [\alpha^{\circ}, \beta\alpha^{\circ}]]$

In detail:

$$\begin{split} & [\beta^{\circ \imath}] \Leftrightarrow (3.) \to (\text{-}2.) \\ & [\text{id}1^{\imath\prime\prime}] \Leftrightarrow (\text{.-}1) \to (\text{.-}1) \\ & [\alpha^{\circ \imath}] \Leftrightarrow (\text{-}2.) \to (1.) \\ & [\beta\alpha^{\prime}] \Leftrightarrow (\text{.-}1) \to (.3) \end{split}$$

Note that using static categorial analysis:

 $(3.1\ 2.1\ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha],$

"complex morphisms" could not be applied, since a possible categorial structure like

 $[\alpha^{\circ}\beta^{\circ}, \alpha', \beta\alpha']$

could be interpreted numerically as (-3.1 - 2.1 - 1.3), (3.-1 2.-1 1.-3), (-3.-1 - 2.-1 - 1.-3) or even as one of the 42 "mixed" sign classes.

Since the idea of introducing negative categories and complex sign classes results from their geometrical display in the Gaussian number field, we shall show, finishing this chapter, in an exemplaric way the 6 transpositions and 6 dual transpositions (reality thematics) of the sign class (3.1 2.1 1.3) in the 4 quadrants, thus getting 48 representation schemes while letting away all "mixed" sign classes for the sake of space. In each graph, the sign classes are solid and the reality thematics dashed:

 $(3.1\ 2.1\ 1.3) \times (3.1\ 1.2\ 1.3)$

 $(3.1\ 1.3\ 2.1) \times (1.2\ 3.1\ 1.3)$







 $(1.3 \ 3.1 \ 2.1) \times (1.2 \ 1.3 \ 3.1)$



(-3.-1 -2.-1 -1.-3) × (-3.-1 -1.-2 -1.-3)



(2.1 1.3 3.1) × (1.3 3.1 1.2)



 $(1.3\ 2.1\ 3.1) \times (1.3\ 1.2\ 3.1)$



$$(-3.-1 - 1.-3 - 2.-1) \times (-1.-2 - 3.-1 - 1.-3)$$







(-3.1 -2.1 -1.3) × (3.-1 1.-2 1.-3)





$$(-1.-3 - 2.-1 - 3.-1) \times (-1.-3 - 1.-2 - 3.-1)$$



(-3.1 -1.3 -2.1) × (1.-2 3.-1 1.-3)



(-2.1 -3.1 -1.3) × (3.-1 1.-3 1.-2)



(-1.3 -3.1 -2.1) × (1.-2 1.-3 3.-1)



(3.-1 2.-1 1.-3) × (-3.1 -1.2 -1.3)



(-2.1 -1.3 -3.1) × (1.-3 3.-1 1.-2)



(-1.3 -2.1 -3.1) × (1.-3 1.-2 3.-1)



(3.-1 1.-3 2.-1) × (-1.2 -3.1 -1.3)



 $(2.-1\ 3.-1\ 1.-3) \times (-3.1\ -1.3\ -1.2)$



 $(2.-1\ 1.-3\ 3.-1) \times (-1.3\ -3.1\ -1.2)$



(1.-3 3.-1 2.-1) × (-1.2 -1.3 -3.1)

(1.-3 2.-1 3.-1) × (-1.3 -1.2 -3.1)



In the next chapter we will show how sign classes and reality thematics are connected with one another, distinguishing between static and dynamic, intra- and trans-semiotic connections and checking out all combinations of sign classes, transpositions and their dualizations, but in order to reduce complexity limiting ourselves to real sign classes.

2. Sign connections

2.1. Static sign connections

2.1.1. Intra-semiotic connections

Each sign class hangs is connected with its reality thematics by at least 1 sub-sign:

1 $(3.1\ 2.1\ \underline{1.1}\ \times\ \underline{1.1}\ 1.2\ 1.3)$ 2 $(3.1 \ \underline{2.1} \ \underline{1.2} \times \underline{2.1} \ \underline{1.2} \ 1.3)$ 3 $(3.1 2.1 1.3 \times 3.1 1.2 1.3)$ 4 $(3.1 \ \underline{2.2} \ 1.2 \times \ 2.1 \ \underline{2.2} \ 1.3)$ 5 $(3.1 2.2 1.3 \times 3.1 2.2 1.3)$ 6 $(3.1 2.3 1.3 \times 3.1 3.2 1.3)$ 7 $(3.2 \underline{2.2} 1.2 \times 2.1 \underline{2.2} 2.3)$ 8 $(3.2 \underline{2.2} 1.3 \times 3.1 \underline{2.2} 2.3)$ $(3.2 2.3 1.3 \times 3.1 3.2 2.3)$ 9 10 $(3.3 2.3 1.3 \times 3.1 3.2 3.3)$

We thus can differentiate between monadically, dyadically and triadically intra-semiotically connected sign classes and reality thematics.

2.1.2. Trans-semiotic connections

Sign classes and reality thematics are connected amongst themselves by 0, 1 or 2 sub-signs. In the following notation "x/y = z" points out that a sign class x is connected with the sign class y by the sub-sign z:

 $\begin{array}{l} 1/2 = 2; 1/3 = 2; 1/4 = 1; 1/5 = 1; 1/6 = 1; 1/7 = 0; 1/8 = 0; 1/9 = 0; 1/10 = 0\\ 2/3 = 2; 2/4 = 2; 2/5 = 1; 2/6 = 1; 2/7 = 1; 2/8 = 0; 2/9 = 0; 2/10 = 0\\ 3/4 = 1; 3/5 = 2; 3/6 = 2; 3/7 = 0; 3/8 = 1; 3/9 = 1; 3/10 = 1\\ 4/5 = 2; 4/6 = 1; 4/7 = 2; 4/8 = 1; 4/9 = 0; 4/10 = 0\\ 5/6 = 2; 5/7 = 1; 5/8 = 2; 5/9 = 1; 5/10 = 1\\ 6/7 = 0; 6/8 = 1; 6/9 = 2; 6/10 = 2\\ 7/8 = 2; 7/9 = 1; 7/10 = 0\\ 8/9 = 2; 8/10 = 1\\ 9/10 = 2\end{array}$

Examples:

```
\begin{array}{l} (3.2 \ 2.2 \ 1.2) \ / \ (3.3 \ 2.3 \ 1.3) = \varnothing \\ (3.2 \ 2.2 \ 1.3) \ / \ (3.3 \ 2.3 \ 1.3) = (1.3) \\ (3.2 \ 2.3 \ 1.3) \ / \ (3.3 \ 2.3 \ 1.3) = (2.3 \ 1.3). \end{array}
```

2.2. Dynamic sign connections

2.2.1. Intra-semiotic connections

Sign classes and their reality thematics can also be connected by identical pairs of sub-signs and thus by semiotic morphisms. In this case all transpositions have to be scrutinized separately.

1	$(3.1\ 2.1\ 1.1 \times 1.1\ 1.2\ 1.3)$	
2	$(3.1 \ \underline{2.1 \ 1.2} \times \underline{2.1 \ 1.2} \ 1.3)$	$(2.1 \rightarrow 1.2)$
3	(3.1 2.1 1.3 × 3.1 1.2 1.3)	
4	$(3.1\ 2.2\ 1.2 \times 2.1\ 2.2\ 1.3)$	
5	$(3.1\ \underline{2.2}\ 1.3 \times \underline{3.1\ 2.2}\ 1.3)$	$(3.1 \rightarrow 2.2) \ (2.2 \rightarrow 1.3)$
6	(3.1 2.3 1.3 × 3.1 3.2 1.3)	
7	$(3.2\ 2.2\ 1.2 \times 2.1\ 2.2\ 2.3)$	
8	$(3.2\ 2.2\ 1.3 \times 3.1\ 2.2\ 2.3)$	
9	(3.2 2.3 1.3 × 3.1 3.2 2.3)	
10	(3.3 2.3 1.3 × 3.1 3.2 3.3)	
1	(311121×121113)	
2	$(3.1 1.2 2.1 \times 1.2 2.1 1.3)$	$(1.2 \rightarrow 2.1)$
3	$(3.1\ 1.3\ 2.1\ \times\ 1.2\ 3.1\ 1.3)$	$(3.1 \rightarrow 1.3)$
4	$(3.1 1.2 2.2 \times 2.2 2.1 1.3)$	
5	$(3.1\ 1.3\ 2.2 \times 2.2\ 3.1\ 1.3)$	$(3.1 \rightarrow 1.3)$
6	$(3.1 1.3 2.3 \times 3.2 3.1 1.3)$	$(3.1 \rightarrow 1.3)$
7	(3.2 1.2 2.2 × 2.2 2.1 2.3)	
8	(3.2 1.3 2.2 × 2.2 3.1 2.3)	
9	(3.2 1.3 2.3 × 3.2 3.1 2.3)	
10	(3.3 1.3 2.3 × 3.2 3.1 3.3)	
1	(2.1 3.1 1.1) × (1.1 1.3 1.2)	
2	$(2.1 \ 3.1 \ 1.2) \times (2.1 \ 1.3 \ 1.2)$	
3	$(2.1, 3.1, 1.3) \times (3.1, 1.3, 1.2)$	$(3.1 \rightarrow 1.3)$
4	$(2.2 \ 3.1 \ 1.2) \times (2.1 \ 1.3 \ 2.2)$	
5	$(2.2 \ 3.1 \ 1.3) \times (3.1 \ 1.3 \ 2.2)$	
6	$(2.3 \ \underline{3.1 \ 1.3}) \times (\underline{3.1 \ 1.3} \ 3.2)$	$(3.1 \rightarrow 1.3)$
7	$(2.2\ 3.2\ 1.2) \times (2.1\ 2.3\ 2.2)$	
8	$(2.2 \ 3.2 \ 1.3) \times (3.1 \ 2.3 \ 2.2)$	
9	(<u>2.3 3.2</u> 1.3) × (3.1 <u>2.3 3.2</u>)	$(2.3 \rightarrow 3.2)$
10	(2.3 3.3 1.3) × (3.1 3.3 3.2)	

Thus the connections of sign classes by semiotic morphisms are non-trivial because they vary between all systems of transpositions. Therefore, in the next chapter we shall show all possible combinations of transpositions and their dualizations (thus including the sign classes and their reality thematics). Recurrent identical morphisms are solid, recurrent inverted morphisms are dotted.

2.2.2. Combinations of transpositions and dual transpositions

Since the occurrence and structure of dyadic and triadic morphismic connections between sign classes and reality thematics are not predictable, we will scrutinize them for each of the 10 sign classes separately, looking for combinations of transpositions, of dual transpositions and of combinations of transpositions and dual transpositions individually.

2.2.2.1. Sign class (3.1 2.1 1.1)

Transpositions vs. transpositions:

3.1 <u>2.1 1.1</u> <u>3.1 2.1 1.1</u> 3.1 <u>2.1 1.1</u> <u>3.1 2.1 1.1</u> <u>3.1 2.1 1.1</u>	3.1 <u>1.1 2.1</u> <u>2.1 3.1</u> 1.1 <u>2.1 1.1</u> 3.1 1.1 <u>3.1 2.1</u> <u>1.1 2.1 3.1</u>	<u>3.1 1.1</u> 2.1 <u>3.1 1.1 2.1</u> <u>3.1 1.1</u> 2.1 3.1 <u>1.1 2.1</u> 3.1 <u>1.1 2.1</u>	2.1 <u>3.1 1.1</u> 2.1 1.1 3.1 <u>1.1 3.1</u> 2.1 <u>1.1 2.1</u> 3.1	2.1 <u>3.1 1.1</u> 2.1 3.1 1.1 2.1 3.1 1.1 2.1 3.1 1.1	2.1 <u>1.1 3.1</u> <u>1.1 3.1 2.1</u> 1.1 <u>2.1 3.1</u>
2.1 <u>1.1 3.1</u> <u>2.1 1.1</u> 3.1	<u>1.1 3.1</u> 2.1 <u>1.1 2.1</u> 3.1	1.1 <u>3.1 2.1</u>	1.1 <u>2.1 3.1</u>		

Dual transpositions vs. dual transpositions:

<u>1.1 1.2 1.3</u> 1.1 <u>1.2 1.3</u> <u>1.1 1.2 1.3</u> 1.1 1.2 1.3	<u>1.2 1.1</u> 1.3 1.1 <u>1.3 1.2</u> 1.3 <u>1.1 1.2</u> 1.2 1.3 1.1	1.2 <u>1.1 1.3</u> <u>1.2 1.1 1.3</u> 1.2 <u>1.1 1.3</u> 1.2 <u>1.1 1.3</u> 1.2 1.1 1.3	<u>1.1 1.3</u> 1.2 <u>1.3 1.1 1.2</u> 1.2 <u>1.3 1.1</u> 1.3 1.2 1.1	<u>1.1 1.3</u> 1.2 <u>1.1 1.3 1.2</u> 1.1 <u>1.3 1.2</u>	<u>1.3 1.1</u> 1.2 <u>1.2 1.3 1.1</u> <u>1.3 1.2</u> 1.1
<u>1.1 1.2 1.3</u>	<u>1.3 1.2 1.1</u>	1.2 1.1 1.3			
<u>1.3 1.1</u> 1.2 1.3 <u>1.1 1.2</u>	1.2 <u>1.3 1.1</u> 1.3 <u>1.2 1.1</u>	<u>1.2 1.3</u> 1.1	<u>1.3 1.2</u> 1.1		

Transpositions vs. dual transpositions:

3.1 2.1 1.1 3.1 2.1 1.1 3.1 2.1 1.1 3.1 2.1 1.1 3.1 2.1 1.1 3.1 2.1 1.1	1.2 1.1 1.3 1.1 1.3 1.2 1.3 1.1 1.2 1.2 1.3 1.1 1 3 1 2 1 3	3.1 1.1 2.1 3.1 1.1 2.1 3.1 1.1 2.1 3.1 1.1 2.1	1.1 1.3 1.2 1.3 1.1 1.2 1.2 1.3 1.1 1.3 1.2 1.1	2.1 3.1 1.1 2.1 3.1 1.1 2.1 3.1 1.1	1.3 1.1 1.2 1.2 1.3 1.1 1.3 1.2 1.1
2.1 1.1 3.1 2.1 1.1 3.1	1.2 1.3 1.1 1.3 1.2 1.1	1.1 3.1 2.1	1.3 1.2 1.1		

2.2.2.2. Sign Class (3.1 2.1 1.2)

Transpositions vs. transpositions:

3.1 <u>2.1 1.2</u> <u>3.1 2.1 1.2</u> 3.1 <u>2.1 1.2</u> <u>3.1 2.1 1.2</u> <u>3.1 2.1 1.2</u> <u>3.1 2.1 1.2</u>	3.1 <u>1.2 2.1</u> <u>2.1 3.1</u> 1.2 <u>2.1 1.2</u> 3.1 1.2 <u>3.1 2.1</u> <u>1.2 2.1 3.1</u>	3.1 1.2 2.1 3.1 1.2 2.1 3.1 1.2 2.1 3.1 1.2 2.1	2.1 <u>3.1 1.2</u> 2.1 1.2 3.1 <u>1.2 3.1</u> 2.1 <u>1.2 2.1</u> 3.1	2.1 <u>3.1 1.2</u> 2.1 3.1 1.2 2.1 3.1 1.2 2.1 3.1 1.2	2.1 <u>1.2 3.1</u> <u>1.2 3.1 2.1</u> 1.2 <u>2.1 3.1</u>
2.1 <u>1.2 3.1</u> <u>2.1 1.2</u> 3.1	<u>1.2 3.1</u> 2.1 <u>1.2 2.1</u> 3.1	1.2 <u>3.1 2.1</u>	1.2 <u>2.1 3.1</u>		

Dual transpositions vs. dual transpositions:

<u>2.1 1.2</u> 1.3	<u>1.2 2.1</u> 1.3	1.2 <u>2.1 1.3</u>	<u>2.1 1.3</u> 1.2	<u>2.1 1.3</u> 1.2	<u>1.3 2.1</u> 1.2
2.1 <u>1.2 1.3</u>	2.1 <u>1.3 1.2</u>	<u>1.2 2.1 1.3</u>	<u>1.3 2.1 1.2</u>	2.1 1.3 1.2	<u>1.2 1.3 2.1</u>
<u>2.1 1.2</u> 1.3	1.3 <u>2.1 1.2</u>	1.2 <u>2.1 1.3</u>	1.2 <u>1.3 2.1</u>	2.1 <u>1.3 1.2</u>	<u>1.3 1.2</u> 2.1
2.1 <u>1.2 1.3</u>	<u>1.2 1.3</u> 2.1	<u>1.2 2.1</u> 1.3	1.3 <u>1.2 2.1</u>		
2.1 1.2 1.3	<u>1.3 1.2 2.1</u>				
<u>1.3 2.1</u> 1.2	1.2 <u>1.3 2.1</u>	<u>1.2 1.3</u> 2.1	<u>1.3 1.2</u> 2.1		
1.3 <u>2.1 1.2</u>	1.3 <u>1.2 2.1</u>				

Transpositions vs. dual transpositions:

3.1 <u>2.1 1.2</u> 3.1 2.1 1.2 3.1 <u>2.1 1.2</u> 3.1 <u>2.1 1.2</u> 3.1 2.1 1.2	<u>1.2 2.1</u> 1.3 2.1 1.3 1.2 1.3 <u>2.1 1.2</u> 1.2 1.3 2.1	3.1 1.2 2.1 3.1 <u>1.2 2.1</u> 3.1 1.2 2.1 3.1 1.2 2.1	2.1 1.3 1.2 1.3 <u>2.1 1.2</u> 1.2 1.3 2.1 1.3 1.2 2.1	2.1 3.1 1.2 2.1 3.1 1.2 2.1 3.1 1.2	1.3 2.1 1.2 1.2 1.3 2.1 1.3 1.2 2.1
3.1 <u>2.1 1.2</u>	1.3 <u>1.2 2.1</u>				
2.1 1.2 3.1 <u>2.1 1.2</u> 3.1	1.2 1.3 2.1 1.3 <u>1.2 2.1</u>	1.2 3.1 2.1	1.3 1.2 2.1		

2.2.2.3. Sign Class (3.1 2.1 1.3)

Transpositions vs. transpositions:

3.1 <u>2.1 1.3</u> <u>3.1 2.1 1.3</u> 3.1 <u>2.1 1.3</u> <u>3.1 2.1 1.3</u> <u>3.1 2.1 1.3</u> 3.1 2.1 1.3	3.1 <u>1.3 2.1</u> <u>2.1 3.1</u> 1.3 <u>2.1 1.3</u> 3.1 1.3 <u>3.1 2.1</u> 1.3 2.1 3.1	<u>3.1 1.3</u> 2.1 <u>3.1 1.3 2.1</u> <u>3.1 1.3</u> 2.1 3.1 <u>1.3 2.1</u>	2.1 <u>3.1 1.3</u> 2.1 1.3 3.1 1.3 3.1 2.1 1.3 2.1 3.1	2.1 <u>3.1 1.3</u> 2.1 3.1 1.3 2.1 3.1 1.3 2.1 3.1 1.3	2.1 <u>1.3 3.1</u> <u>1.3 3.1 2.1</u> 1.3 <u>2.1 3.1</u>
2.1 <u>1.3 3.1</u> <u>2.1 1.3</u> 3.1	<u>1.3 3.1</u> 2.1 <u>1.3 2.1</u> 3.1	1.3 <u>3.1 2.1</u>	1.3 <u>2.1 3.1</u>		

Dual transpositions vs. dual transpositions:

3.1 1.2 1.3 3.1 1.2 1.3 3.1 1.2 1.3 3.1 1.2 1.3 3.1 1.2 1.3	<u>1.2 3.1</u> 1.3 3.1 <u>1.3 1.2</u> 1.3 <u>3.1 1.2</u> <u>1.2 1.3</u> 3.1	1.2 <u>3.1 1.3</u> <u>1.2 3.1 1.3</u> 1.2 <u>3.1 1.3</u> <u>1.2 3.1 1.3</u> <u>1.2 3.1</u> 1.3	3.1 1.3 1.2 1.3 3.1 1.2 1.2 <u>1.3 3.1</u> 1.3 <u>1.2 3.1</u>	<u>3.1 1.3</u> 1.2 <u>3.1 1.3 1.2</u> 3.1 <u>1.3 1.2</u> 3.1 <u>1.3 1.2</u>	<u>1.3 3.1</u> 1.2 <u>1.2 1.3 3.1</u> <u>1.3 1.2</u> 3.1
<u>3.1.1.2.1.3</u> <u>1.3.3.1</u> 1.2 1.3.3.1 1.2	<u>1.3 1.2 3.1</u> 1.2 <u>1.3 3.1</u> 1.3 1.2 3.1	<u>1.2 1.3</u> 3.1	<u>1.3 1.2</u> 3.1		

Transpositions vs. dual transpositions:

3.1 2.1 1.3	1.2 3.1 1.3	<u>3.1 1.3</u> 2.1	<u>3.1 1.3</u> 1.2	2.1 3.1 1.3	<u>1.3 3.1</u> 1.2
3.1 2.1 1.3	3.1 1.3 1.2	<u>3.1 1.3</u> 2.1	<u>1.3 3.1</u> 1.2	2.1 <u>3.1 1.3</u>	1.2 <u>1.3 3.1</u>
3.1 2.1 1.3	1.3 3.1 1.2	<u>3.1 1.3</u> 2.1	1.2 <u>1.3 3.1</u>	2.1 3.1 1.3	1.3 1.2 3.1
3.1 2.1 1.3	1.2 1.3 3.1	3.1 1.3 2.1	1.3 1.2 3.1		
3.1 2.1 1.3	1.3 1.2 3.1				
2.1 <u>1.3 3.1</u>	1.2 <u>1.3 3.1</u>	1.3 3.1 2.1	1.3 1.2 3.1		
2.1 1.3 3.1	1.3 1.2 3.1				

2.2.2.4. Sign Class (3.1 2.2 1.2)

Transpositions vs. transpositions:

3.1 <u>2.2 1.2</u> <u>3.1 2.2 1.2</u> 3.1 <u>2.2 1.2</u> <u>3.1 2.2 1.2</u> <u>3.1 2.2 1.2</u> <u>3.1 2.2 1.2</u>	3.1 <u>1.2 2.2</u> <u>2.2 3.1</u> 1.2 <u>2.2 1.2</u> 3.1 1.2 <u>3.1 2.2</u> 1.2 2.2 3.1	<u>3.1 1.2 2.2</u> <u>3.1 1.2 2.2</u> <u>3.1 1.2 2.2</u> <u>3.1 1.2 2.2</u> 3.1 <u>1.2 2.2</u>	2.2 <u>3.1 1.2</u> <u>2.2 1.2 3.1</u> <u>1.2 3.1</u> 2.2 <u>1.2 2.2</u> 3.1	2.2 <u>3.1 1.2</u> 2.2 <u>3.1 1.2</u> 2.2 <u>3.1</u> 1.2	2.2 <u>1.2 3.1</u> <u>1.2 3.1 2.2</u> 1.2 <u>2.2 3.1</u>
2.2 <u>1.2 3.1</u> 2.2 <u>1.2</u> 3.1	<u>1.2 3.1</u> 2.2 <u>1.2 2.2</u> 3.1	1.2 <u>3.1 2.2</u>	1.2 <u>2.2 3.1</u>		

Dual transpositions vs. dual transpositions:

<u>2.1 2.2</u> 1.3	<u>2.2 2.1</u> 1.3	2.2 <u>2.1 1.3</u>	<u>2.1 1.3</u> 2.2	<u>2.1 1.3</u> 2.2	<u>1.3 2.1</u> 2.2
2.1 <u>2.2 1.3</u>	2.1 <u>1.3 2.2</u>	<u>2.2.2.1.1.3</u>	<u>1.3 2.1 2.2</u>	2.1 <u>1.3 2.2</u>	<u>2.2 1.3</u> 2.1
<u>2.1 2.2</u> 1.3	1.3 <u>2.1 2.2</u>	2.2 <u>2.1 1.3</u>	2.2 <u>1.3 2.1</u>	2.1 <u>1.3 2.2</u>	<u>1.3 2.2</u> 2.1
2.1 <u>2.2 1.3</u>	<u>2.2 1.3</u> 2.1	<u>2.2 2.1</u> 1.3	1.3 <u>2.2 2.1</u>		
2.1 2.2 1.3	1.3 2.2 2.1				
<u>1.3 2.1</u> 2.2	2.2 <u>1.3 2.1</u>	<u>2.2 1.3</u> 2.1	<u>1.3 2.2</u> 2.1		
<u>1.3 2.1</u> 2.2	2.2 <u>1.3 2.1</u>				

Transpositions vs. dual transpositions:

3.1 2.2 1.2	2.2 2.1 1.3	3.1 1.2 2.2	2.1 1.3 2.2	2.2 3.1 1.2	1.3 2.1 2.2
3.1 2.2 1.2	2.1 1.3 2.2	3.1 1.2 2.2	1.3 2.1 2.2	2.2 3.1 1.2	2.2 1.3 2.1
3.1 2.2 1.2	1.3 2.1 2.2	3.1 1.2 2.2	2.2 1.3 2.1	2.2 3.1 1.2	1.3 2.2 2.1
3.1 2.2 1.2	2.2 1.3 2.1	3.1 1.2 2.2	1.3 2.2 2.1		
3.1 2.2 1.2	1.3 2.2 2.1				
2.2 1.2 3.1	2.2 1.3 2.1	1.2 3.1 2.2	1.3 2.2 2.1		
2.2 1.2 3.1	1.3 2.2 2.1				

2.2.2.5. Sign Class (3.1 2.2 1.3)

Transpositions vs. Transpositions:

3.1 <u>2.2 1.3</u> 3 1 2 2 1 3	3.1 <u>1.3 2.2</u> 2 2 3 1 1 3	3.1 1.3 2.2	2.2 <u>3.1 1.3</u> 2 2 1 3 3 1	$\begin{array}{c c} 2.2 & \underline{3.1 \ 1.3} \\ 2.2 & \underline{3.1 \ 1.3} \\ \end{array}$	2.2 <u>1.3 3.1</u>
<u>3.1 2.2 1.3</u> 3.1 <u>2.2 1.3</u>	$\frac{2.2}{2.2}\frac{5.1}{1.3}$ 3.1	<u>3.1 1.3 2.2</u> <u>3.1 1.3</u> 2.2	<u>2.2 1.3 3.1</u> <u>1.3 3.1</u> 2.2	<u>2.2 3.1 1.3</u> <u>2.2 3.1</u> 1.3	1.3 <u>2.2 3.1</u>
<u>3.1 2.2</u> 1.3 <u>3.1 2.2 1.3</u>	1.3 <u>3.1 2.2</u> 1.3 2.2 3.1	3.1 <u>1.3 2.2</u>	<u>1.3 2.2</u> 3.1		
2.2 1.3 3.1	1.3 3.1 2.2	1.3 3.1 2.2	1.3 2.2 3.1		
<u>2.2 1.3</u> 3.1	<u>1.3 2.2</u> 3.1				

Dual transpositions vs. dual transpositions:

<u>3.1 2.2</u> 1.3	<u>2.2 3.1</u> 1.3	2.2 <u>3.1 1.3</u>	<u>3.1 1.3</u> 2.2	<u>3.1 1.3</u> 2.2	<u>1.3 3.1</u> 2.2
3.1 <u>2.2 1.3</u>	3.1 <u>1.3 2.2</u>	<u>2.2 3.1 1.3</u>	<u>1.3 3.1 2.2</u>	<u>3.1 1.3 2.2</u>	<u>2.2 1.3 3.1</u>
<u>3.1 2.2</u> 1.3	1.3 <u>3.1 2.2</u>	2.2 <u>3.1 1.3</u>	2.2 <u>1.3 3.1</u>	3.1 <u>1.3 2.2</u>	<u>1.3 2.2</u> 3.1
3.1 <u>2.2 1.3</u>	<u>2.2 1.3</u> 3.1	<u>2.2 3.1</u> 1.3	1.3 <u>2.2 3.1</u>		
3.1 2.2 1.3	1.3 2.2 3.1				
<u>1.3 3.1</u> 2.2	2.2 <u>1.3 3.1</u>	<u>2.2 1.3</u> 3.1	<u>1.3 2.2</u> 3.1		
1.3 <u>3.1 2.2</u>	1.3 <u>2.2 3.1</u>				

Transpositions vs. dual transpositions:

<u>3.1 2.2</u> 1.3	<u>2.2 3.1</u> 1.3	<u>3.1 1.3 2.2</u>	<u>3.1 1.3 2.2</u>	2.2 3.1 1.3	<u>1.3 3.1 2.2</u>
3.1 <u>2.2 1.3</u>	3.1 <u>1.3 2.2</u>	<u>3.1 1.3</u> 2.2	<u>1.3 3.1</u> 2.2	2.2 <u>3.1 1.3</u>	2.2 <u>1.3 3.1</u>
<u>3.1 2.2</u> 1.3	1.3 <u>3.1 2.2</u>	3.1 1.3 2.2	<u>2.2 1.3 3.1</u>	<u>2.2 3.1</u> 1.3	1.3 <u>2.2 3.1</u>
3.1 <u>2.2 1.3</u>	<u>2.2 1.3</u> 3.1	3.1 <u>1.3 2.2</u>	<u>1.3 2.2</u> 3.1		
<u>3.1 2.2 1.3</u>	<u>1.3 2.2 3.1</u>				
<u>2.2 1.3 3.1</u>	<u>2.2 1.3 3.1</u>	1.3 <u>3.1 2.2</u>	1.3 <u>2.2 3.1</u>		
<u>2.2 1.3</u> 3.1	<u>1.3 2.2</u> 3.1				

2.2.2.6. Sign Class (3.1 2.3 1.3)

Transpositions vs. transpositions:

3.1 <u>2.3 1.3</u> <u>3.1 2.3 1.3</u> 3.1 <u>2.3 1.3</u> <u>3.1 2.3 1.3</u> <u>3.1 2.3 1.3</u> <u>3.1 2.3 1.3</u>	3.1 <u>1.3.2.3</u> <u>2.3 3.1</u> 1.3 <u>2.3 1.3</u> 3.1 1.3 <u>3.1 2.3</u> <u>1.3 2.3 3.1</u>	3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3	2.3 <u>3.1 1.3</u> <u>2.3 1.3 3.1</u> <u>1.3 3.1</u> 2.3 <u>1.3 2.3</u> 3.1	2.3 <u>3.1 1.3</u> <u>2.3 3.1 1.3</u> <u>2.3 3.1</u> 1.3	2.3 <u>1.3 3.1</u> <u>1.3 3.1 2.3</u> 1.3 <u>2.3 3.1</u>
2.3 <u>1.3 3.1</u> <u>2.3 1.3</u> 3.1	<u>1.3 3.1</u> 2.3 <u>1.3 2.3</u> 3.1	1.3 <u>3.1 2.3</u>	1.3 <u>2.3 3.1</u>		

Dual transpositions vs. dual transpositions:

<u>3.1 3.2 1.3</u>	<u>3.2 3.1</u> 1.3	3.2 <u>3.1 1.3</u>	<u>3.1 1.3</u> 3.2	3.1 1.3 3.2	<u>1.3 3.1</u> 3.2
3.1 <u>3.2 1.3</u> 3.1 3.2 1.3	3.1 <u>1.3 3.2</u> 1.3 3.1 3.2	<u>3.2 3.1 1.3</u> 3.2 3.1 1.3	<u>1.3 3.1 3.2</u> 3.2 1.3 3.1	<u>3.1 1.3 3.2</u> 3.1 1.3 3.2	<u>3.2 1.3 3.1</u> 1.3 3.2 3.1
3.1 <u>3.2</u> 1.3	<u>3.2 1.3</u> 3.1	<u>3.2 3.1</u> 1.3	1.3 <u>3.2 3.1</u>		
<u>3.1 3.2 1.3</u>	<u>1.3 3.2 3.1</u>				
<u>1.3 3.1</u> 3.2	3.2 <u>1.3 3.1</u>	<u>3.2 1.3</u> 3.1	<u>1.3 3.2</u> 3.1		
1.3 <u>3.1 3.2</u>	1.3 <u>3.2 3.1</u>				

Transpositions vs. dual transpositions:

3.1 2.3 1.3 3.1 2.3 1.3 3.1 2.3 1.3 3.1 2.3 1.3 3.1 2.3 1.3 3.1 2.3 1.3	3.2 3.1 1.3 3.1 1.3 3.2 1.3 3.1 3.2 3.2 1.3 3.1 1.3 3.2	3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3 3.1 1.3 2.3	<u>3.1 1.3</u> 3.2 <u>1.3 3.1</u> 3.2 3.2 <u>1.3 3.1</u> 1.3 3.2 3.1	2.3 <u>3.1 1.3</u> 2.3 <u>3.1 1.3</u> 2.3 3.1 1.3	<u>1.3 3.1</u> 3.2 3.2 <u>1.3 3.1</u> 1.3 3.2 3.1
2.3 <u>1.3 3.1</u> 1.3 <u>3.1 3.2</u>	3.2 <u>1.3 3.1</u> 1.3 <u>3.2 3.1</u>	1.3 3.1 2.3	1.3 3.2 3.1		

2.2.2.7. Sign Class (3.2 2.2 1.2)

Transpositions vs. transpositions:

3.2 <u>2.2 1.2</u>	3.2 <u>1.2 2.2</u>	<u>3.2 1.2</u> 2.2	2.2 <u>3.2 1.2</u>	2.2 3.2 1.2	2.2 <u>1.2 3.2</u>
<u>3.2 2.2</u> 1.2	<u>2.2 3.2</u> 1.2	<u>3.2 1.2 2.2</u>	<u>2.2 1.2 3.2</u>	2.2.3.2.1.2	<u>1.2 3.2 2.2</u>
3.2 <u>2.2 1.2</u>	<u>2.2 1.2</u> 3.2	<u>3.2 1.2</u> 2.2	<u>1.2 3.2</u> 2.2	<u>2.2 3.2</u> 1.2	1.2 <u>2.2 3.2</u>
<u>3.2 2.2</u> 1.2	1.2 <u>3.2 2.2</u>	3.2 <u>1.2 2.2</u>	<u>1.2 2.2</u> 3.2		
3.2 2.2 1.2	1.2 2.2 3.2				
2.2 <u>1.2 3.2</u>	<u>1.2 3.2</u> 2.2	1.2 <u>3.2 2.2</u>	1.2 <u>2.2 3.2</u>		
<u>2.2 1.2</u> 3.2	<u>1.2 2.2</u> 3.2				

Dual transpositions vs. dual transpositions:

2.1 2.2 2.3 2.1 2.2 2.3 2.1 2.2 2.3 2.1 2.2 2.3 2.1 2.2 2.3	2.2 2.1 2.3 2.1 2.3 2.2 2.3 2.1 2.2 2.2 2.3 2.1 2.2 2.3 2.1	2.2 <u>2.1 2.3</u> <u>2.2 2.1 2.3</u> 2.2 <u>2.1 2.3</u> <u>2.2 2.1 2.3</u> <u>2.2 2.1</u> 2.3	2.1 2.3 2.2 2.3 2.1 2.2 2.2 2.3 2.1 2.3 <u>2.2 2.1</u>	<u>2.1 2.3</u> 2.2 <u>2.1 2.3 2.2</u> 2.1 <u>2.3 2.2</u> 2.1 <u>2.3 2.2</u>	<u>2.3 2.1</u> 2.2 <u>2.2 2.3 2.1</u> <u>2.3 2.2</u> 2.1
<u>2.3 2.1</u> 2.2 2.3 <u>2.1</u> 2.2 2.3 <u>2.1 2.2</u>	2.2 <u>2.3 2.1</u> 2.2 <u>2.3 2.1</u> 2.3 <u>2.2 2.1</u>	<u>2.2 2.3</u> 2.1	<u>2.3 2.2</u> 2.1		

Transpositions vs. dual transpositions:

3.2 2.2 1.2	2.2 2.1 2.3	3.2 1.2 2.2	2.1 2.3 2.2	2.2 3.2 1.2	2.3 2.1 2.2
3.2 2.2 1.2	2.1 2.3 2.2	3.2 1.2 2.2	2.3 2.1 2.2	2.2 3.2 1.2	2.2 2.3 2.1
3.2 2.2 1.2	2.3 2.1 2.2	3.2 1.2 2.2	2.2 2.3 2.1	2.2 3.2 1.2	2.3 2.2 2.1
3.2 2.2 1.2	2.2 2.3 2.1	3.2 1.2 2.2	2.3 2.2 2.1		
3.2 2.2 1.2	2.3 2.2 2.1				
2.2 1.2 3.2	2.2 2.3 2.1	1.2 3.2 2.2	2.3 2.2 2.1		
2.2 1.2 3.2	2.3 2.2 2.1				

2.2.2.8. Sign Class (3.2 2.2 1.3)

Transpositions vs. transpositions:

3.2 <u>2.2 1.3</u>	3.2 <u>1.3 2.2</u>	<u>3.2 1.3</u> 2.2	2.2 <u>3.2 1.3</u>	2.2 3.2 1.3	2.2 <u>1.3 3.2</u>
<u>3.2 2.2</u> 1.3	<u>2.2 3.2</u> 1.3	3.2 1.3 2.2	<u>2.2 1.3 3.2</u>	2.2 3.2 1.3	<u>1.3 3.2 2.2</u>
3.2 <u>2.2 1.3</u>	<u>2.2 1.3</u> 3.2	<u>3.2 1.3</u> 2.2	<u>1.3 3.2</u> 2.2	<u>2.2 3.2</u> 1.3	1.3 <u>2.2 3.2</u>
<u>3.2 2.2</u> 1.3	1.3 <u>3.2 2.2</u>	3.2 <u>1.3 2.2</u>	<u>1.3 2.2</u> 3.2		
<u>3.2 2.2 1.3</u>	<u>1.3 2.2 3.2</u>				
2.2 <u>1.3 3.2</u>	<u>1.3 3.2</u> 2.2	1.3 <u>3.2 2.2</u>	1.3 <u>2.2 3.2</u>		
<u>2.2 1.3</u> 3.2	<u>1.3 2.2</u> 3.2				

Dual transpositions vs. dual transpositions:

3.1 2.2 2.3 3.1 2.2 2.3 3.1 2.2 2.3 3.1 2.2 2.3 3.1 2.2 2.3 3.1 2.2 2.3	2.2.3.1 2.3 3.1 2.3 2.2 2.3 <u>3.1 2.2</u> 2.2 2.3 3.1 2.3 2.2 3.1	2.2 <u>3.1 2.3</u> 2.2 <u>3.1 2.3</u> 2.2 <u>3.1 2.3</u> 2.2 <u>3.1 2.3</u> <u>2.2 3.1</u> 2.3	3.1 2.3 2.2 2.3 3.1 2.2 2.2 2.3 3.1 2.3 2.2 3.1	<u>3.1 2.3</u> 2.2 <u>3.1 2.3 2.2</u> 3.1 <u>2.3 2.2</u> 3.1 <u>2.3 2.2</u>	<u>2.3 3.1</u> 2.2 <u>2.2 2.3 3.1</u> <u>2.3 2.2</u> 3.1
<u>2.3 3.1</u> 2.2 2.3 <u>3.1 2.2</u>	2.2 <u>2.3 3.1</u> 2.3 <u>2.2 3.1</u>	<u>2.2 2.3</u> 3.1	<u>2.3 2.2</u> 3.1		
Transpositions vs. dual transpositions:

3.2 2.2 1.3 3.2 2.2 1.3	2.2 3.1 2.3 3.1 2.3 2.2	3.2 1.3 2.2 3.2 1.3 2.2	3.1 2.3 2.2 2.3 3.1 2.2	2.2 3.2 1.3 2.2 3.2 1.3	2.3 3.1 2.2 2.2 2.3 3.1
3.2 2.2 1.3 3.2 2.2 1.3 3.2 2.2 1.3	2.3 3.1 2.2 2.2 2.3 3.1 2 3 2 2 3 1	3.2 1.3 2.2 3.2 1.3 2.2	2.2 2.3 3.1 2.3 2.2 3.1	2.2 3.2 1.3	2.3 2.2 3.1
2.2 1.3 3.2 2.2 1.3 3.2	2.2 2.3 3.1 2.3 2.2 3.1	1.3 3.2 2.2	2.3 2.2 3.1		

2.2.2.9. Sign Class (3.2 2.3 1.3)

Transpositions vs. transpositions:

3.2 <u>2.3 1.3</u>	3.2 <u>1.3 2.3</u>	<u>3.2 1.3</u> 2.3	2.3 <u>3.2 1.3</u>	2.3 3.2 1.3	2.3 <u>1.3</u> <u>3.2</u>
<u>3.2 2.3</u> 1.3	<u>2.3 3.2</u> 1.3	3.2 1.3 2.3	<u>2.3 1.3 3.2</u>	2.3 3.2 1.3	<u>1.3 3.2 2.3</u>
3.2 <u>2.3 1.3</u>	<u>2.3 1.3</u> 3.2	<u>3.2 1.3 2.3</u>	<u>1.3 3.2</u> 2.3	<u>2.3 3.2</u> 1.3	1.3 <u>2.3 3.2</u>
<u>3.2 2.3</u> 1.3	1.3 <u>3.2 2.3</u>	3.2 <u>1.3 2.3</u>	<u>1.3 2.3</u> 3.2		
3.2 2.3 1.3	<u>1.3 2.3 3.2</u>				
231332	133223	133223	132332		
<u>2.3 1.3</u> 3.2	<u>1.3 2.3</u> 2.3 <u>1.3 2.3</u> 3.2		1.5 <u>2.5 5.2</u>		

Dual transpositions vs. dual transpositions:

<u>3.1 3.2</u> 2.3	<u>3.2 3.1</u> 2.3	3.2 <u>3.1 2.3</u>	<u>3.1 2.3</u> 3.2	<u>3.1 2.3</u> 3.2	<u>2.3 3.1</u> 3.2
3.1 <u>3.2 2.3</u>	3.1 <u>2.3 3.2</u>	<u>3.2 3.1 2.3</u>	<u>2.3 3.1 3.2</u>	<u>3.1 2.3 3.2</u>	<u>3.2 2.3 3.1</u>
<u>3.1 3.2</u> 2.3	2.3 <u>3.1 3.2</u>	3.2 <u>3.1 2.3</u>	3.2 <u>2.3 3.1</u>	3.1 <u>2.3 3.2</u>	<u>2.3 3.2</u> 3.1
3.1 <u>3.2 2.3</u>	<u>3.2 2.3</u> 3.1	<u>3.2 3.1</u> 2.3	2.3 <u>3.2 3.1</u>		
<u>3.1 3.2 2.3</u>	2.3 3.2 3.1				
<u>2.3 3.1</u> 3.2	3.2 <u>2.3 3.1</u>	<u>3.2 2.3</u> 3.1	<u>2.3 3.2</u> 3.1		
2.3 <u>3.1 3.2</u>	2.3 <u>3.2 3.1</u>				

Transpositions vs. dual transpositions:

3.2 2.3 1.3	3.2 3.1 2.3	3.2 1.3 2.3	3.1 2.3 3.2	2.3 3.2 1.3	2.3 3.1 3.2
<u>3.2 2.3</u> 1.3	3.1 <u>2.3 3.2</u>	3.2 1.3 2.3	2.3 3.1 3.2	<u>2.3 3.2</u> 1.3	<u>3.2 2.3</u> 3.1
3.2 2.3 1.3	2.3 3.1 3.2	3.2 1.3 2.3	3.2 2.3 3.1	<u>2.3 3.2</u> 1.3	<u>2.3 3.2</u> 3.1
<u>3.2 2.3</u> 1.3	<u>3.2 2.3</u> 3.1	3.2 1.3 2.3	2.3 3.2 3.1		
<u>3.2 2.3</u> 1.3	<u>2.3 3.2</u> 3.1				
2 3 1 3 3 2	3 7 7 3 3 1	133773	733731		
2.5 1.5 5.2	5.2 2.5 5.1	1.5 <u>5.2 2.5</u>	<u>2.3 J.2</u> J.1		
2.3 1.3 3.2	2.3 3.2 3.1				

2.2.2.10. Sign Class (3.3 2.3 1.3)

Transpositions vs. transpositions:

3.3 <u>2.3 1.3</u> <u>3.3 2.3</u> 1.3	3.3 <u>1.3 2.3</u> <u>2.3 3.3</u> 1.3	$\begin{array}{c c} \underline{3.3 \ 1.3} \ 2.3 \\ \underline{3.3 \ 1.3} \ 2.3 \\ \end{array}$	2.3 <u>3.3 1.3</u> <u>2.3 1.3 3.3</u>	2.3 <u>3.3 1.3</u> 2.3 <u>3.3 1.3</u>	2.3 <u>1.3 3.3</u> <u>1.3 3.3 2.3</u>
3.3 <u>2.3 1.3</u> <u>3.3 2.3</u> 1.3 <u>3.3 2.3 1.3</u>	<u>2.3 1.3</u> 3.3 1.3 <u>3.3 2.3</u> <u>1.3 2.3 3.3</u>	<u>3.3 1.3</u> 2.3 3.3 <u>1.3 2.3</u>	<u>1.3 3.3</u> 2.3 <u>1.3 2.3</u> 3.3	<u>2.3 3.3</u> 1.3	1.3 <u>2.3 3.3</u>
2.3 <u>1.3 3.3</u> <u>2.3 1.3</u> 3.3	<u>1.3 3.3</u> 2.3 <u>1.3 2.3</u> 3.3	1.3 <u>3.3 2.3</u>	1.3 <u>2.3 3.3</u>		

Dual transpositions vs. dual transpositions:

<u>3.1 3.2</u> 3.3	<u>3.2 3.1</u> 3.3	3.2 <u>3.1 3.3</u>	<u>3.1 3.3</u> 3.2	<u>3.1 3.3</u> 3.2	<u>3.3 3.1</u> 3.2
3.1 3.2 3.3	3.1 <u>3.3 3.2</u>	<u>3.2 3.1 3.3</u>	<u>3.3 3.1 3.2</u>	3.1 3.3 3.2	<u>3.2 3.3 3.1</u>
<u>5.1 5.2</u> 5.5 3 1 3 2 3 3	3.3 <u>3.1 3.2</u> 3.2 3 3 3 1	3.2 <u>3.1 3.3</u>	3.2 <u>3.3 3.1</u> 3 3 3 2 3 1	3.1 <u>3.3 3.2</u>	<u>3.3 3.2</u> 3.1
<u>3.1 3.2 3.3</u>	<u>5.2 5.5</u> 5.1 3.3 3.2 3.1	<u>5.2 5.1</u> 5.5	5.5 <u>5.2 5.1</u>		
<u>3.3 3.1</u> 3.2 3.3 <u>3.1 3.2</u>	3.2 <u>3.3 3.1</u> 3.3 <u>3.2 3.1</u>	<u>3.2 3.3</u> 3.1	<u>3.3 3.2</u> 3.1		

Transpositions vs. dual transpositions:

3.3 2.3 1.3	3.2 3.1 3.3	3.3 1.3 2.3	3.1 3.3 3.2	2.3 3.3 1.3	3.3 3.1 3.2
3.3 2.3 1.3	3.1 3.3 3.2	3.3 1.3 2.3	3.3 3.1 3.2	2.3 3.3 1.3	3.2 3.3 3.1
3.3 2.3 1.3	3.3 3.1 3.2	3.3 1.3 2.3	3.2 3.3 3.1	2.3 3.3 1.3	3.3 3.2 3.1
3.3 2.3 1.3	3.2 3.3 3.1	3.3 1.3 2.3	3.3 3.2 3.1		
3.3 2.3 1.3	3.3 3.2 3.1				
2.3 1.3 3.3	3.2 3.3 3.1	1.3 3.3 2.3	3.3 3.2 3.1		
2.3 1.3 3.3	3.3 3.2 3.1				

Besides the 10 sign classes we will look at the connecting structures of the transpositions of the Genuine Category Class (main diagonal of the semiotic matrix) whose formal affinities to the dual-invariant sign class (3.1 2.2 1.3) had been pointed out by Bense (1992, pp. 34 ss., 52 ss.).

2.2.2.11. Genuine Category Class (3.3 2.2 1.1)

3.3 <u>2.2 1.1</u>	3.3 <u>1.1 2.2</u>	<u>3.3 1.1</u> 2.2	2.2 <u>3.3 1.1</u>	2.2 3.3 1.1	2.2 <u>1.1 3.3</u>
<u>3.3 2.2</u> 1.1	<u>2.2 3.3</u> 1.1	3.3 1.1 2.2	<u>2.2 1.1 3.3</u>	<u>2.2 3.3 1.1</u>	1.1 3.3 2.2
3.3 <u>2.2 1.1</u>	<u>2.2 1.1</u> 3.3	3.3 1.1 2.2	<u>1.1 3.3</u> 2.2	<u>2.2 3.3</u> 1.1	1.1 <u>2.2 3.3</u>
<u>3.3 2.2</u> 1.1	1.1 <u>3.3 2.2</u>	3.3 <u>1.1 2.2</u>	<u>1.1 2.2</u> 3.3		
<u>3.3 2.2 1.1</u>	<u>1.1 2.2 3.3</u>				
2.2 <u>1.1 3.3</u>	<u>1.1 3.3</u> 2.2	1.1 3.3 2.2	1.1 <u>2.2 3.3</u>		
<u>2.2 1.1</u> 3.3	<u>1.1 2.2</u> 3.3				

Transpositions vs. transpositions:

Dual transpositions vs. dual transpositions:

<u>1.1 2.2 3.3</u> 1.1 <u>2.2 3.3</u> <u>1.1 2.2 3.3</u> 1.1 <u>2.2 3.3</u> 1.1 <u>2.2 3.3</u>	2.2 1.1 3.3 1.1 <u>3.3 2.2</u> 3.3 <u>1.1 2.2</u> 2.2 <u>3.3</u> 1.1 <u>3.3 2.2 1.1</u>	$\begin{array}{c} 2.2 \ \underline{1.1} \ \underline{3.3} \\ \underline{2.2} \ \underline{1.1} \ \underline{3.3} \end{array}$	<u>1.1 3.3</u> 2.2 <u>3.3 1.1 2.2</u> 2.2 <u>3.3 1.1</u> 3.3 <u>2.2 1.1</u>	<u>1.1 3.3</u> 2.2 <u>1.1 3.3 2.2</u> 1.1 <u>3.3 2.2</u>	<u>3.3 1.1</u> 2.2 <u>2.2 3.3 1.1</u> <u>3.3 2.2</u> 1.1
<u>3.3 1.1</u> 2.2 3.3 <u>1.1 2.2</u>	2.2 <u>3.3 1.1</u> 3.3 <u>2.2 1.1</u>	<u>2.2 3.3</u> 1.1	<u>3.3 2.2</u> 1.1		

Transpositions vs. dual transpositions:

3.3 <u>2.2 1.1</u> <u>3.3 2.2</u> 1.1 <u>3.3 2.2 1.1</u> <u>3.3 2.2 1.1</u> <u>3.3 2.2 1.1</u>	<u>2.2 1.1</u> 3.3 1.1 <u>3.3 2.2</u> 3.3 <u>1.1 2.2</u> <u>2.2 3.3</u> 1.1	$\begin{array}{r} 3.3 1.1 2.2 \\ \underline{3.3 1.1 2.2} \\ \underline{3.3 1.1 2.2} \\ \underline{3.3 1.1 2.2} \\ 3.3 \underline{1.1 2.2} \end{array}$	<u>1.1 3.3 2.2</u> <u>3.3 1.1 2.2</u> 2.2 <u>3.3 1.1</u> 3.3 <u>2.2 1.1</u>	$\begin{array}{r} 2.2 \ \underline{3.3} \ \underline{1.1} \\ \underline{2.2} \ \underline{3.3} \ \underline{1.1} \\ \underline{2.2} \ \underline{3.3} \ \underline{1.1} \\ \underline{2.2} \ \underline{3.3} \ \underline{1.1} \end{array}$	<u>3.3 1.1</u> 2.2 <u>2.2 3.3 1.1</u> <u>3.3 2.2</u> 1.1
<u>2.2 1.1 3.3</u> <u>2.2 1.1</u> 3.3	2.2 <u>3.3 1.1</u> 3.3 <u>2.2 1.1</u>	1.1 <u>3.3 2.2</u>	<u>3.3 2.2</u> 1.1		

As one recognizes, all combinations of transpositions (sign classes and reality thematics) obey the following scheme:

			1	
 right		left		right
 left		triadic-inverted		triadic-inverted
 right		left		left
 left		right		
 triadic-inverted		5		

 right	 right
 left	

The pattern of the combinations of dual transpositions amongst themselves is the same, except that the positions of the semiotic morphisms are mirrored, i.e. inverted:

 left	—	_	right		left
 right			triadic-inverted		triadic-inverted
 left			right	—	right
 right	_	-	left		
 triadic-inverted					
 left			left		
 right					

In the combinations of transpositions and dual transpositions there is no constant pattern. But because of their several symmetries it is worth to have a look at the patterns of the dual-invariant sign class (3.1 2.2 1.3) and the Genuine Category Class (3.3 2.2 1.1).

The dual-invariant sign class shows the following pattern:

 left		triadic	 triadic-inverted
 right		left	 right
 left		triadic-inverted	 left
 right		right	
 triadic-inverted		-	
 triadic		right	
 left		_	

The pattern of the Genuine Category Class looks as follows:

 right		left	—	left
 left		triadic		triadic
 right		left		left
 left		right		
 triadic				
 right		right		
 left		_		

Thus the two patterns are completely different. In the next chapter, we will use the static and dynamic intra- and trans-semiotic sign connections established here in order to show the network structure of semiotic ghost trains.

3. Semiotic ghost trains

3.1. Transpositional realities

As we know, each sign class can be transformed into its reality thematic by inversion of both the order of the sub-signs and of the sub-signs themselves, this semiotic operation being called dualization (while triadic values become trichotomic and vice versa). Thus reality thematics are considered to present the **structural reality** of each sign class, insofar as in all but one reality thematics of the system of the 10 sign classes two sub-signs of the same triadic value determine a sub-sign of the same or of different values, thus presenting dyadic reality. However, there are only three sign classes where all three sub-signs of the reality thematics have the same triadic value:

 $\begin{array}{l} (3.1 \ 2.1 \ 1.1) \times (1.1 \ \underline{1.2 \ 1.3}) \\ (3.2 \ 2.2 \ 1.2) \times (2.1 \ \underline{2.2 \ 2.3}) \\ (3.3 \ 2.3 \ 1.3) \times (3.1 \ \underline{3.2 \ 3.3}) \end{array}$

They appear as rows of the semiotic matrix and are called main sign classes, since they are trichotomically homogeneous. The only sign class whose structural reality is admitted to be not dyadic but triadic is the dual-invariant sign class:

 $(3.1\ 2.2\ 1.3) \times (\underline{3.1\ 2.2\ 1.3}),$

which exhibits or "thematizes" three structural realities:

(2.2 1.3)-thematized (3.1) (3.1 1.3)-thematized (2.2) (3.1 2.2)-thematized (1.3),

while the reality thematics of the other six sign classes are assumed to present each only one structural reality:

$(3.2\ 2.2\ 1.3) \times (3.1\ \underline{2.2\ 2.3})$:	(2.2 2.3)-thematized (3.1)
$(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3})$:	(1.2 1.3)-thematized (3.1)
$(3.1\ 2.1\ 1.2) \times (2.1\ \underline{1.2\ 1.3})$:	(1.2 1.3)-thematized (2.1)
$(3.2\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 2.3):$	(3.1 3.2)-thematized (2.3)
$(3.1\ 2.2\ 1.2) \times (\underline{2.1\ 2.2}\ 1.3)$:	(2.1 2.2)-thematized (1.3)
$(3.1\ 2.3\ 1.3) \times (\underline{3.1\ 3.2}\ 1.3)$:	(3.1 3.2)-thematized (1.3)

In addition to the dual-invariant sign class, the inverse-invariant Genuine Category Class (3.3 2.2 1.1) × (1.1 2.2 3.3) also presents a triadic structural reality and thus three thematizations:

(2.2 1.1)-thematized (3.3) (3.3 1.1)-thematized (2.2) (3.3 2.2)-thematized (1.1).

But if we have a look at the transpositions of the reality thematics of each sign class, we recognize that each reality thematic exhibits a threefold structural reality, may it be triadic or dyadic. As an example, we shall present here the transpositions of the reality thematic of the sign class (3.1 2.1 1.3):

$(3.1\ 2.1\ 1.3) \times (3.1\ 1.2\ 1.3)$:	(3.1 1.2)-thematized (1.3) (3.1 1.3)-thematized (1.2) (1.2 1.3)-thematized (3.1)
$(3.1\ 1.3\ 2.1) \times (\underline{1.2\ 3.1\ 1.3})$:	(1.2 3.1)-thematized (1.3) (1.2 1.3)-thematized (3.1) (3.1 1.3)-thematized (1.2)
(2.1 3.1 1.3) × (<u>3.1 1.3 1.2</u>):	(3.1 1.3)-thematized (1.2) (3.1 1.2)-themazited (1.3) (1.3 1.2)-thematized (3.1)
(2.1 1.3 3.1) × (<u>1.3 3.1 1.2</u>):	(1.3 3.1)-thematized (1.2) (1.3 1.2)-thematized (3.1) (3.1 1.2)-thematized (1.3)
$(1.3 \ 3.1 \ 2.1) \times (\underline{1.2} \ \underline{1.3} \ \underline{3.1}):$	(1.2 1.3)-thematized (3.1) (1.2 3.1)-thematized (1.3) (1.3 3.1)-thematized (1.2)
$(1.3\ 2.1\ 3.1) \times (\underline{1.3\ 1.2\ 3.1})$:	(1.3 1.2)-thematized (3.1) (1.3 3.1)-thematized (1.2) (1.2 3.1)-thematized (1.3)

The triadic structural realities of each reality thematic are connected to each other by shared reality thematics; from the standpoint of the whole system of structural realities – but not from the part-systems -, 12 of the 18 structural realities are redundant. We therefore can sum up the structural realities in 6 types. In order to point out these 6 types clearly, we introduce a general notation in which the "basis" displays the triadic value of a thematizing or thematized sub-sign, the "exponent" the frequency of this sub-sign and the arrow the direction of thematization. For instance, the reality thematic (3.1 <u>1.2 1.3</u>) can be noted as $3^1 \leftarrow 1^2$, the left-handed arrow indicating that the sub-sign with triadic value 1 appears with frequency 2 and thus thematizes the sub-sign with triadic value 3 whose frequency is only 1. Thus we get the following structural realities for the transpositions of the reality thematic of the sign class (3.1 2.1 1.3):

If we compare the six structural realities presented by each sign class, we recognize that besides the "regular" right-left-thematizations:

$$(3.1\ 2.1\ 1.3) \times (3.1\ \underline{1.2\ 1.3}):$$
 $3^1 \leftarrow 1^2$

there are left-right-thematizations:

$$(1.3\ 2.1\ 3.1) \times (\underline{1.3\ 1.2\ }3.1): \qquad 1^2 \to 3^1$$

inside of both the right-left as well as the left-right-thematizations the order, i.e. the placevalue of the two thematizing sub-signs plays a role:

(3.1 2.1 (2.1 3.1	$1.3) \times (3.1 \ \underline{1.2 \ 1.3}):$ $1.3) \times (3.1 \ \underline{1.3 \ 1.2}):$	$3^{1} \leftarrow 1^{2}$ $3^{1} \leftarrow 1^{2}$
(1.3 3.1 (1.3 2.1	$2.1) \times (\underline{1.2 \ 1.3} \ 3.1):$ $3.1) \times (\underline{1.3 \ 1.2} \ 3.1):$	$1^2 \rightarrow 3^1$ $1^2 \rightarrow 3^1$

we find so-called "sandwich-thematizations" (cf. Toth 2007, p. 216), in which the order to the thematizing sub-signs plays a role, too:

(3.1 1.3 2.1) × (<u>1.2</u> 3.1 <u>1.3</u>):	$1^1 \leftarrow 3^1 \rightarrow 1^1$
(2.1 1.3 3.1) × (<u>1.3 3.1 1.2</u>):	$1^1 \leftarrow 3^1 \rightarrow 1^1$

However, our notation does apparently not yet allow differentiating between types of thematizations whose thematizing sub-signs are distinguished by different place-values. We may help this by introducing the signs "<" and ">" in juxtaposition to the "exponents":

(3.1 <u>1.2 1.3</u>):	$3^1 \leftarrow 1^{2,<}$
(<u>1.2</u> 3.1 <u>1.3</u>):	$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$
(3.1 <u>1.3 1.2</u>):	$3^1 \leftarrow 1^{2,>}$
(<u>1.3</u> 3.1 <u>1.2</u>):	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$
(<u>1.2 1.3</u> 3.1):	$1^{2,<} \rightarrow 3^1$
(<u>1.3 1.2</u> 3.1):	$1^{2,>} \rightarrow 3^1$

If we also want to cope with complex sign classes, we may further introduce the negative algebraic sign, which we may either put in front of the "basis" or the "exponent". We thus get the following complete reality theoretic structure for the sign class (3.1 2.1 1.3):

(3.1 <u>1.2 1.3</u>):	$3^1 \leftarrow 1^{2,<}$	(31 <u>12 13</u>):	$3^{-1} \leftarrow 1^{-2,<}$
(<u>1.2</u> 3.1 <u>1.3</u>):	$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$	(<u>12</u> 31 <u>13</u>):	$1^{-1,<} \rightarrow 3^{-1} \leftarrow 1^{-1}$
(3.1 <u>1.3 1.2</u>):	$3^1 \leftarrow 1^{2,>}$	(31 <u>13 12</u>):	$3^{-1} \leftarrow 1^{-2,>}$
(<u>1.3</u> 3.1 <u>1.2</u>):	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$	(<u>13</u> 31 <u>12</u>):	$1^{-1,>} \rightarrow 3^{-1} \leftarrow 1^{-1}$
(<u>1.2 1.3</u> 3.1):	$1^{2,<} \leftarrow 3^1$	(<u>12 13</u> 31):	$1^{-2,<} \leftarrow 3^{-1}$
(<u>1.3 1.2</u> 3.1):	$1^{2,>} \leftarrow 3^1$	(<u>13 12</u> 31):	$1^{-2,>} \leftarrow 3^{-1}$
(-3.1 <u>-1.2 -1.3</u>):	$-3^1 \leftarrow -1^{2,<}$	(-31 <u>-12 -13</u>):	$-3^{-1} \leftarrow -1^{-2,<}$
(-3.1 <u>-1.2 -1.3</u>): (<u>-1.2</u> -3.1 <u>-1.3</u>):	$-3^{1} \leftarrow -1^{2,<}$ $-1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1}$	(-31 <u>-12</u> -13): (<u>-12</u> -31 <u>-13</u>):	$-3^{-1} \leftarrow -1^{-2,<}$ $-1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1}$
(-3.1 <u>-1.2 -1.3</u>): (<u>-1.2</u> -3.1 <u>-1.3</u>): (-3.1 <u>-1.3 -1.2</u>):	$-3^{1} \leftarrow -1^{2,<}$ $-1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1}$ $-3^{1} \leftarrow -1^{2,>}$	(-31 <u>-12 -13</u>): (<u>-12</u> -31 <u>-13</u>): (-31 <u>-13 -12</u>):	$-3^{-1} \leftarrow -1^{-2,<} \\ -1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -3^{-1} \leftarrow -1^{-2,>}$
(-3.1 <u>-1.2 -1.3</u>): (<u>-1.2</u> -3.1 <u>-1.3</u>): (-3.1 <u>-1.3 -1.2</u>): (<u>-1.3</u> -3.1 <u>-1.2</u>):	$\begin{array}{c} -3^{1} \leftarrow -1^{2,<} \\ -1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1} \\ -3^{1} \leftarrow -1^{2,>} \\ -1^{1,>} \rightarrow -3^{1} \leftarrow -1^{1} \end{array}$	(-31 <u>-12 -13</u>): (<u>-12</u> -31 <u>-13</u>): (-31 <u>-13 -12</u>): (<u>-13</u> -31 <u>-12</u>):	$\begin{array}{c} -3^{-1} \leftarrow -1^{-2,<} \\ -1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -3^{-1} \leftarrow -1^{-2,>} \\ -1^{-1,>} \rightarrow -3^{-1} \leftarrow -1^{-1} \end{array}$
$(-3.1 \ \underline{-1.2} \ -1.3):$ $(-3.1 \ \underline{-1.3} \ -1.3):$ $(-3.1 \ \underline{-1.3} \ -1.2):$ $(\underline{-1.3} \ -3.1 \ \underline{-1.2}):$ $(\underline{-1.2} \ -3.1):$	$-3^{1} \leftarrow -1^{2,<}$ $-1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1}$ $-3^{1} \leftarrow -1^{2,>}$ $-1^{1,>} \rightarrow -3^{1} \leftarrow -1^{1}$ $-1^{2,<} \leftarrow -3^{1}$	(-31 <u>-12 -13</u>): (<u>-12</u> -31 <u>-13</u>): (-31 <u>-13 -12</u>): (<u>-13</u> -31 <u>-12</u>): (<u>-12 -13</u> -31):	$\begin{array}{c} -3^{-1} \leftarrow -1^{-2,<} \\ -1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -3^{-1} \leftarrow -1^{-2,>} \\ -1^{-1,>} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -1^{-2,<} \leftarrow -3^{-1} \end{array}$
$\begin{array}{l} (-3.1 \ \underline{-1.2} \ -1.3):\\ (\underline{-1.2} \ -3.1 \ \underline{-1.3}):\\ (-3.1 \ \underline{-1.3} \ -1.2):\\ (\underline{-1.3} \ -3.1 \ \underline{-1.2}):\\ (\underline{-1.2} \ -1.3 \ -3.1):\\ (\underline{-1.3} \ -1.2 \ -3.1): \end{array}$	$\begin{array}{c} -3^{1} \leftarrow -1^{2,<} \\ -1^{1,<} \rightarrow -3^{1} \leftarrow -1^{1} \\ -3^{1} \leftarrow -1^{2,>} \\ -1^{1,>} \rightarrow -3^{1} \leftarrow -1^{1} \\ -1^{2,<} \leftarrow -3^{1} \\ -1^{2,>} \leftarrow -3^{1} \end{array}$	$(-31 \ \underline{-12} \ -13):$ $(\underline{-12} \ -31 \ \underline{-13}):$ $(-31 \ \underline{-13} \ -12):$ $(\underline{-13} \ -31 \ \underline{-12}):$ $(\underline{-12} \ -31):$ $(\underline{-13} \ -12 \ -31):$	$\begin{array}{c} -3^{-1} \leftarrow -1^{-2,<} \\ -1^{-1,<} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -3^{-1} \leftarrow -1^{-2,>} \\ -1^{-1,>} \rightarrow -3^{-1} \leftarrow -1^{-1} \\ -1^{-2,<} \leftarrow -3^{-1} \\ -1^{-2,>} \leftarrow -3^{-1} \end{array}$

Using the abstract scheme for reality thematics (f.e d.c b.a), we get the following abstract system of structural realities:

- 1. $(\pm f.\pm e \leftarrow \pm d.\pm c, \pm b.\pm a)$
- 2. $(\pm f.\pm e \leftarrow \pm b.\pm a, \pm d.\pm c)$
- 3. $(\pm d.\pm c, \pm b.\pm a \rightarrow \pm f.\pm e)$
- 4. $(\pm b.\pm a, \pm d.\pm c \rightarrow \pm f.\pm e)$
- 5. $(\pm d.\pm c \rightarrow \pm f.\pm e \leftarrow \pm b.\pm a)$
- 6. $(\pm b.\pm a \rightarrow \pm f.\pm e \leftarrow \pm d.\pm c)$

3.2. The semiotic ghosts

On the basis of a "cosmological topology" current cosmological research assumes a tetrahedral model of the universe: "Represent T as a set G of quaternions acting by conjugation. Now let the same set G act on S³ by multiplication. There is our group Γ of fixed-point free symmetries of the 3-sphere. The only catch is that each of the original symmetries of S² is realized by two different quaternions **q** and –**q** so the group G has twice as many elements as the original group. In the present example with the original group being the tetrahedral group T the final group Γ is the binary tetrahedral group T* of order 24" (Weeks 2004, p. 615). "If the speed of light were infinite inhabitants of the binary tetrahedral space S³/T* would see 24 images of every cosmological object" (Weeks 2004, p. 614).

The mentioned geometrical conditions are fulfilled by a tetrahedral dipyramid, which is shown here to the left as Johnson solid and to the right as folded up two-dimensional model:



(http://mathworld.wolfram.com/TriangularDipyramid.html)

Especially the folded-up model shows that 6 triangles come together, which display in three dimensions a tetrahedral dipyramid. Therefore, the model to the right can be used without loss of generality for the representation of a sign class or a reality thematic with each of their 6 transpositions.

Let us now have a look at the relation of cosmological "objects" and their "ghosts": "The unique image of the object which lies inside the fundamental cell and thus coincides with the original object is called 'real' " (Lachièze-Rey 2003, p. 76). "This 'real part' of the universal covering the basic cell is generally chosen to coincide with the fundamental polyhedron centered on the observer" (Lachièze-Rey 2003, p. 93). In other words: In cosmology, reality is defined as closeness to the observer. However, since the observer can change his standpoint, every object closest to him is real while all other objects observed or observable by him are automatically turned into ghost images of this object, thus totally 24, and this number coincides with the 6 times 4 transpositions of a sign class or reality thematic in the complex number field (cf. Chapter 1.5). Moreover, the sign classes and reality thematics "deformed" in the shape of the transpositions obviously even correspond to the cosmological objects deformed by the effect of the distribution of density: "Because the Universe is not exactly homogeneous, the null geodesics are not exactly those of the spatially homogeneous spacetime. They are deformed by the density inhomogenities leading to the various consequences of gravitational lensing: deformation, amplification, multiplications of images ... A ghost so amplified or distorted may become hard to recognize" (Lachièze-Rev 2003, p. 96).

In Chapter 2, we had seen that sign classes and reality thematics can hang together in the following ways:

statically: by 0, 1 or 2 sub-signs

dynamically: dyadically (left or right position), triadically-inverted or triadically-dualinvariant

In addition to these semiotic connection laws there is the law of determinant-symmetric duality systems found by Walther (1982, p. 18) which states that all 10 sign classes and reality thematics are connected with the dual-invariant sign class (3.1 2.2 1.3) by at least 1 sub-sign:



Thus, while for a static connection just one vertex of the folded-up dipyramid is enough, both the static-dyadic and the dynamic-dyadic connections require edges of the dipyramid. Hence triadic connections are possible only *inside* of a dipyramid. Furthermore, the sign classes and reality thematics correspond to the topological chirality of the dipyramid according to the 6 possible transpositions and the dynamic left and right positions, respectively:



Thus, wherever the observer stands in this lattice of semiotic-topological dipyramids, only that object is "real" to him which is represented by the sign class or reality thematic nearby him, and he thus sees, according to the topological structure and orientation of the semiotic dipyramidal lattice, from each object also the 24 ghosts of this object that he has to perceive as unreal because of the cosmological identification of reality and closeness. It must be clear, however, that on the basis of the identification of reality with closeness under the possibility of free exchange of the observer's standpoint, the notions of "real" and "unreal" become obsolete. Since according to Peirce we cannot perceive and communicate without signs, we therefore exist in a semiotic mirror-maze, which is, amazingly enough, topologically identical with the presently most accepted model of the shape of the universe. This gives a lot of evidence for the conclusion that the semiotic structures of our brain and the physical structure of this universe are basically identical.

Assuming that an observer's position coincides with the reality thematic of a certain sign class, the other 5 transpositions of this reality thematic therefore must appear to him as ghost images of the semiotic object presented in the structural reality of this reality thematic.

Hence the 5 ghost images of each semiotic object may be called "semiotic ghosts" and can be classified according to the types of structural realities presented in the respective reality thematics. In order to show that, we shall take again the sign class (3.1 2.1 1.3). If we focus on the real semiotic reality thematics, we get the following 6 reality thematics with their structural realities:

(3.1 <u>1.2 1.3</u>):	$3^1 \leftarrow 1^{2,<}$
(<u>1.2</u> 3.1 <u>1.3</u>):	$1^{1,<} \rightarrow 3^1 \leftarrow 1^1$
(3.1 <u>1.3 1.2</u>):	$3^1 \leftarrow 1^{2,>}$
(<u>1.3</u> 3.1 <u>1.2</u>):	$1^{1,>} \rightarrow 3^1 \leftarrow 1^1$
(<u>1.2 1.3</u> 3.1):	$1^{2,<} \leftarrow 3^1$
(<u>1.3 1.2</u> 3.1):	$1^{2,>} \leftarrow 3^1$

Since both the object and its ghosts are depending on the observer's standpoint, we need a cybernetic classification compatible with the semiotic realities and their logical counterparts. Günther (1976, pp. 336 ss.) proposed a triadic logical model consisting of an objective subject (oS), an (objective) object (O) and a subjective subject (sS) which I succeeded to identify with the semiotic medium (M), the semiotic object (O) and the semiotic interpretant (I) in this corresponding order (Toth 2008, pp. 64 ss.). In addition, (O, oS) can be identified with the "system" and (sS) with its "environment" (cf. Günther 1979, pp. 215 ss.) which allows to determine an observer in a pure semiotic way. For our sign class (3.1 2.1 1.3) we thus get the following correspondences:

Sign class: (3.1 2.1 1.3) reality thematic: (3.1 1.2 1.3) structural reality: (3.1 <u>1.2 1.3</u>) semiotic: (1.2 1.3)-thematized (3.1) logical: (oS)-thematized (sS) cybernetic: (object-environment / environment-object)-thematized subject

Now we shall watch the behavior of this structural reality in the transpositions of the respective reality thematic and classify the thematizations according to adjacency of the thamatizing sub-signs and to semiosic orientation:

1.3 3.1 2.1×1.2 1.3 3.1 adjacent generative right-oriented oS sS Ο oS1 oS2 sS $oS \rightarrow sS$ $sS \rightarrow oS2$ $O \rightarrow oS1$ 1.3 2.1 3.1×1.3 1.2 3.1 adjacent degenerative right-oriented oS oS1 sS Ο sS oS2 $oS \rightarrow sS$ $O \rightarrow oS2$ $sS \rightarrow oS1$ 2.1 × <u>1.2</u> _ 3.1 3.1 non-adjacent generative medium-oriented 1.3 1.3sS oS Ο oS1 sS oS2 $sS \rightarrow oS2$ $oS \rightarrow sS$ $O \rightarrow oS1$ 2.1 1.3 3.1 × <u>1.3</u> _ 3.1 1.2non-adjacent degenerative medium-oriented Ο sS ---oS1 sS oS2 oS $O \rightarrow oS2$ $oS \rightarrow sS$ $sS \rightarrow oS1$

Therefore, there are the following semiotic-logical types of thematizations, which are valid for all reality thematics:

$M \rightarrow I$	$oS \rightarrow$	sS
$O \rightarrow M1, M2$	$\mathrm{O} \rightarrow$	oS1, oS2
$I \rightarrow M1, M2$	$sS \rightarrow$	oS1, oS2

Since the cybernetic system consists of the semiotic M and O or of the logical oS and O, respectively, in the above scheme, therefore, only the semiotic and the logical object are constant insofar as they cannot appear to the right side of the arrows and stand in an exchange relation only with the objective subject. In other words: Object and subjective subject are never exchanged in transpositions, i.e. the cybernetic difference between system and environment is always kept up. However, the objective subject, standing in exchange relation with the (objective) object, can itself stand in exchange relation with the subjective subject. This indirect cyclic relation between M, O and I or oS, O and sS, respectively, guaranteed by *two* objective subjects but only *one* object and *one* subjective subject for each structural reality thus enables the observer to take place in each position of the 6 reality thematics inside of the semiotic dipyramid, which thus leads to an also cyclic exchange between semiotic objects and ghosts. In other words: What is considered a semiotic ghost and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition "unreal" and what is considered a semiotic object and thus by definition the observer who can, as we have already seen, take plac

in all 6 positions. Thus, not only is the dichotomy between "real" and "unreal" abolished and replaced by a scale of different degrees of "reality" and "unreality"; the difference itself leads ad absurdum, since each object can become a ghost and each ghost can become an object, since the different structural realities in the system stand in 6! = 720 exchange relations in a system of 6 real transpositions and in $24! = 6.20448402 \times 10^{23}$ exchange relations in a system 24 complex transpositions. In the following, we shall therefore show the 24 complex ghosts of the sign class (3.1 2.1 1.3) together with their coordinate reality thematics, each pair of mutually dual transpositions in one graph:

 $(3.1\ 2.1\ 1.3 \times 3.1\ \underline{1.2\ 1.3})$







 $(3.-1\ 2.-1\ 1.-3 \times -3.1\ -\underline{1.2\ -1.3})$









(2.-1 3.-1 1.-3 × -3.1 -<u>1.3 -1.2</u>)



 $(1.3 \ 3.1 \ 2.1 \times \underline{1.2 \ 1.3} \ 3.1)$





(-2.-1 -3.-1 -1.-3 × -3.-1 -<u>1.-3 -1.-2</u>)



(-1.3 -3.1 -2.1 × <u>1.-2 1.-3</u> 3.-1)



(1.-3 3.-1 2.-1 × -<u>1.2 -1.3</u> -3.1)



 $(1.3\ 2.1\ 3.1 \times \underline{1.3\ 1.2}\ 3.1)$



 $(1.-3\ 2.-1\ 3.-1\times -\underline{1.3\ -1.2}\ -3.1)$



(-1.-3 -3.-1 -2.-1 × -<u>1.-2 -1.-3</u> -3.-1)



(-1.3 -2.1 -3.1 × <u>1.-3 1.-2</u> 3.-1)



(-1.-3 -2.-1 -3.-1 × -<u>1.-3 -1.-2</u> -3.-1)





(3.-1 1.-3 2.-1 × -<u>1.2</u> -3.1 -<u>1.3</u>)



(2.1 1.3 3.1 × <u>1.3</u> 3.1 <u>1.2</u>)



(-3.1 -1.3 -2.1 × <u>1.-2</u> 3.-1 <u>1.-3</u>)



(-3.-1 -1.-3 -2.-1 × -<u>1.-2</u> -3.-1 -<u>1.-3</u>)



(-2.1 -1.3 -3.1 × <u>1.-3</u> 3.-1 <u>1.-2</u>)



 $(2.-1\ 1.-3\ 3.-1\times -\underline{1.3}\ -3.1\ -\underline{1.2})$

(-2.-1 -1.-3 -3.-1 × -<u>1.-3</u> -3.-1 -<u>1.-2</u>)



In the next chapter, we shall investigate the tracks that connect the different types of semiotic ghosts, thus establishing some fragments of semiotic ghost trains, highly complex networks of connecting and transitional paths between semiotic objects and their ghost images.

3.3. The semiotic rails

3.3.1. Semiotic bigraphs

The differentiation between static and dynamic categorial analysis in Chapter 1.3 also allows differentiating between "locality" and "connectivity" in sign classes and reality thematics, a conceptual device upon which "bigraphs" have been recently introduced in computer-based mathematics. Thus each bipartite graphs displays two independent structures upon a given set of nodes, called "place graph" and "link graph", which may be connected to one another by nodes called "ports" (Milner 2008). We remember that each sign class can be turned into a twofold categorial notation:

 $(3.1 \ 2.1 \ 1.3) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$ $(3.1 \ 2.1 \ 1.3) \Leftrightarrow ((3.1 \ 2.1) \ (2.1 \ 1.3)) \equiv [[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]],$

whereby the dynamic analysis takes into account the intra-semiotic connections, i.e. in the abstract categorial scheme

[[W, X], [Y, Z]],

where W, ..., Z stand for categories, the respective sub-signs

[[a.b c.d], [c.d e.f]]

are mapped onto their dynamic categories in the following way:

 $(a.c) \Leftrightarrow [W], (b.d) \Leftrightarrow [X]; (c.e) \Leftrightarrow [Y], (d.f) \Leftrightarrow [Z],$

thus taking notice of the fact that each triadic sign class can be understood as latticetheoretic intersection of two dyadic sign relations (Walther 1979, p. 79); cf.

 $\begin{array}{l} (3.1 \ 2.1) \ (2.1 \ 1.3) \Leftrightarrow [[\beta^{\circ}, \operatorname{id}1], [\alpha^{\circ}, \beta\alpha]] \neq \\ (3.1 \ 2.1) \ (2.1 \ 1.3) \Leftrightarrow [[\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}], [\alpha^{\circ}, \beta\alpha]] = [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha] = (3.1 \ 2.1 \ 1.3) \end{array}$

In the following we shall show that static categorial analysis refers to the locality of a sign relation, while dynamic categorial analysis refers to their connectivity. Thus the interaction of locality and connectivity and the possible intersection of their sub-graphs or sets of sub-signs in ports is a new way to show sign connections in completion to the results obtained in Chapter 2. We first show the bigraphic connections of the 10 sign classes:

	locality	connectivity	port-nodes
3.1 2.1 1.1	[α°β°, <u>α°</u> , <u>id1]</u>	[β°, <u>id1], [α°, id1]</u>	[α° , id1]
3.1 2.1 1.2	$[\alpha^{\circ}\beta^{\circ}, \underline{\alpha^{\circ}}, \underline{\alpha}]$	[β°, id1], [<u>α°</u> , <u>α</u>]	[α°, α]
3.1 2.1 1.3	[α°β°, <u>α°, βα]</u>	[β°, id1], [<u>α°</u> , <u>βα</u>]	[α°, βα]
3.1 2.2 1.2	[α°β°, <u>id2</u> , <u>α]</u>	[β°, <u>α], [</u> α°, <u>id2]</u>	[id2, α]
3.1 2.2 1.3	[α°β° , id2, β α]	$[\beta^{\circ}, \alpha], [\alpha^{\circ}, \beta]$	Ø
3.1 2.3 1.3	[α°β°, β, <u>βα]</u>	[β°, <u>βα]</u> , [α°, id3]	[βα]
3.2 2.2 1.2	[<u>β°</u> , <u>id2</u> , α]	[<u>β°, id2]</u> , [α°, <u>id2]</u>	[β° , id2]
3.2 2.2 1.3	[<u>β°</u> , <u>id2</u> , βα]	$[\underline{\beta^{\circ}}, \underline{id2}], [\alpha^{\circ}, \beta]$	[β° , id2]
3.2 2.3 1.3	[<u>β°, β</u> , βα]	[<u>β°, β], [</u> α°, id3]	[β°, β]
3.3 2.3 1.3	$[\underline{id3}, \beta, \beta\alpha]$	[β°, <u>id3]</u> , [α°, <u>id3]</u>	[id3]
3.3 2.2 1.1	[id3, id2, id1]	$[\beta^{\circ}, \beta^{\circ}], [\alpha^{\circ}, \alpha^{\circ}]$	Ø

One thus recognizes that there are sign classes, which are bigraphically connected only with the left, only with the right or with both sides of their semiotic hypergraphs. There are forests whose nodes are connected with two nodes of their hypergraphs. Only the dual-invariant sign class (3.1 2.2 1.3) and the Genuine Category Class (3.3 2.2 1.1) have empty port-nodes, and there is thus no connection between their forests and hypergraphs.

We shall now display the interaction of locality and connectivity and the possible existence of port-nodes in the system of the transpositions and dual transpositions of the sign class (3.1 2.1 1.3):

	locality	connectivity	port-nodes
3.1 2.1 1.3	[α°β°, <u>α°, βα]</u>	[β°, id1], [<u>α°, βα]</u>	[α°, βα]
3.1 1.3 2.1	[<u>α°β°, βα</u> , α°]	[[<u>α°β°</u> , <u>βα]</u> , [α, <u>α°β°]]</u>	$[\alpha^{\circ}\beta^{\circ},\beta\alpha]$
2.1 3.1 1.3	[α°, <u>α°β°</u> , <u>βα]</u>	[[β, id1], [<u>α°β°, βα]]</u>	[α°β°, βα]
2.1 1.3 3.1	$[\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}]$	[[<u>α°</u> , <u>βα],</u> [βα, <u>α°β°]]</u>	$[\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}]$
1.3 3.1 2.1	[<u>βα, α°β°</u> , α°]	[[<u>βα, α°β°]</u> , [β°, id1]]	[βα, α°β°]
1.3 2.1 3.1	[βα, α°, <u>α°β°]</u>	$[[\alpha, \underline{\alpha^{\circ}\beta^{\circ}}], [\beta, id1]]$	[α°β°]
3.1 1.2 1.3	<u>[α°β°, α</u> , βα]	[[<u>α°β°</u> , <u>α], [id1, β]]</u>	[α°β°, α]
1.2 3.1 1.3	[α, <u>α°β°, βα]</u>	[[βα, α°], [<u>α°β°, βα]]</u>	[α°β°, βα]
3.1 1.3 1.2	[<u>α°β°, βα</u> , α]	[[<u>α°β°</u> , <u>βα]</u> , [id1, β°]]	$[\alpha^{\circ}\beta^{\circ},\beta\alpha]$
1.3 3.1 1.2	[<u>βα, α°β°, α]</u>	[[<u>βα</u> , <u>α°β°</u>], <u>[α°β°</u> , <u>α]]</u>	$[\beta \alpha, \alpha^{\circ} \beta^{\circ}, \alpha]$
1.2 1.3 3.1	$[\alpha, \underline{\beta\alpha}, \underline{\alpha^{\circ}\beta^{\circ}}]$	[[id1, β], [<u>βα, α°β°]]</u>	[βα, α°β°]
1.3 1.2 3.1	$[\underline{\beta\alpha}, \alpha, \alpha^{\circ}\beta^{\circ}]$	[[id1, β°], [<u>βα</u> , α°]]	[βα]

Apparently, the numerical transpositions, their categorial locality, their categorial connectivity and their port-nodes are dual as the sign classes are to their reality thematics:

(3.1 2.1 1.3)	\rightarrow	[α°, βα]	X	[α°β°, α]	\rightarrow	(3.1 1.2 1.3)
(3.1 1.3 2.1)	\rightarrow	[α°β°, βα]	×	[α°β° , βα]	\rightarrow	(1.2 3.1 1.3)
(2.1 3.1 1.3)	\rightarrow	[α°β°, βα]	X	[α°β° , βα]	\rightarrow	(3.1 1.3 1.2)
(2.1 1.3 3.1)	\rightarrow	$[\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}]$] X	$[\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha]$	\rightarrow	(1.3 3.1 1.2)
(1.3 3.1 2.1)	\rightarrow	[βα, α°β°]	×	[βα, α°β°]	\rightarrow	(1.2 1.3 3.1)
(1.3 2.1 3.1)	\rightarrow	[α°β°]	X	[βα]	\rightarrow	(1.3 1.2 3.1)

Moreover, the transpositions (3.1 1.3 2.1) and (2.1 3.1 1.3) have the same port-nodes which they share with their dual transpositions (1.2 3.1 1.3) and (3.1 1.3 1.2) due to the fact that their categorial port-node structure $[\alpha^{\circ}\beta^{\circ},\beta\alpha]$ is self-dual.

Therefore, semiotic bigraphs turn out to be a useful instrument for showing connections between sign structures that are more intricate than the ones shown in Chapter 2. We thus propose a new graphic model for semiotic bigraphs in order to visualize semiotic connectedness using the transpositions of the sign class (3.1 2.1 1.3). The rectangles to the left are the place graphs, the ones to the right the link graphs and the lines between the categorial sub-signs are connecting the ports:

Semiotic bigraph for (3.1 2.1 1.3):



Semiotic bigraph for (3.1 1.3 2.1):



Semiotic bigraph for (2.1 3.1 1.3):



Semiotic bigraph for (2.1 1.3 3.1):



Semiotic bigraph for (1.3 3.1 2.1):



Semiotic bigraph for (1.3 2.1 3.1):



In the following, we will show the connections between some transpositions of the sign class (3.1 2.1 1.3) and its reality thematic using both the static and the dynamic-bigraphical model. As a result, the different types of connections are mostly different, the bigraphical method thus disclosing otherwise hidden semiotic connections. As we shall see, it is further necessary to differentiate between three geometrical types of connections (straight; detour; intersectional) and between mono- and poly-connectedness:



Let us now have a closer look at the different kinds of semiotic connections:

1. (3.1 2.1 1.3) / (3.1 1.2 1.3):

Numerical: Static categorial:	$(3.1 \ 2.1 \ 1.3) \cap (3.1 \ 1.2 \ 1.3) = (3.1, 1.3)$ $[\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha] \cap [\alpha^{\circ}\beta^{\circ}, \alpha, \beta\alpha] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha]$ Type: straight/mono-connected
Dynamic categorial:	[[β° , id1], [α° , $\beta\alpha$]] \cap [[$\alpha^{\circ}\beta^{\circ}$, α], [id1, β]] = [id1] = (1.1) Type: detour/mono-connected
Ports:	$[\alpha^{\circ},\beta\alpha] \cap [\alpha^{\circ}\beta^{\circ},\alpha] = \emptyset$
2. (3.1 1.2 1.3) / (2.1	3.1 1.3):
Numerical: Static categorial:	$(3.1\ 1.2\ 1.3) \cap (2.1\ 3.1\ 1.3) = (3.1, 1.3)$ $[\alpha^{\circ}\beta^{\circ}, \alpha, \beta\alpha] \cap [\alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha]$ Type: detour/mono-connected; straight/mono-connected
Dynamic categorial:	$[[\alpha^{\circ}\beta^{\circ}, \alpha], [id1, \beta]] \cap [[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] = [\alpha^{\circ}\beta^{\circ}, id1] \equiv (3.1, 1.1)$ Type: intersectional/mono-connected
Ports:	$[\alpha^{\circ}\beta^{\circ}, \alpha] \cap [\alpha^{\circ}, \beta\alpha] = \emptyset$
3. (2.1 3.1 1.3) / (1.3	3.1 1.2):
Numerical: Static categorial:	$(2.1 \ 3.1 \ 1.3) \cap (1.3 \ 3.1 \ 1.2) = (3.1, 1.3)$ $[\alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha] \cap [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha]$ Type: intersectional/mono-connected
Dynamic categorial:	[[β , id1], [$\alpha^{\circ}\beta^{\circ}$, $\beta\alpha$]] \cap [[$\beta\alpha$, $\alpha^{\circ}\beta^{\circ}$], [$\alpha^{\circ}\beta^{\circ}$, α]] = [$\alpha^{\circ}\beta^{\circ}$, $\beta\alpha$] = (3.1, 1.3)
Ports:	Type: intersectional/poly-connected $[\alpha^{\circ}, \beta\alpha] \cap [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha] = [\beta\alpha] \equiv (1.3)$ Type: detour/mono-connected
4. (1.3 3.1 1.2) / (1.3	1.2 3.1):
Numerical: Static categorial:	$(1.3 \ 3.1 \ 1.2) \cap (1.3 \ 1.2 \ 3.1) = (1.3, 3.1, 1.2)$ $[\beta \alpha, \alpha^{\circ} \beta^{\circ}, \alpha] \cap [\beta \alpha, \alpha, \alpha^{\circ} \beta^{\circ}] = [\beta \alpha, \alpha^{\circ} \beta^{\circ}, \alpha]$ Type: straight/mono-connected: intersectional/mono-connected
Dynamic categorial:	[[$\beta\alpha, \alpha^{\circ}\beta^{\circ}$], [$\alpha^{\circ}\beta^{\circ}, \alpha$]] \cap [[id1, β°], [$\beta\alpha, \alpha^{\circ}$]] = [$\beta\alpha$] \equiv (1.3) Type: detour/mono-connected
Ports:	$[\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha] \cap [\beta\alpha] = [\beta\alpha] \equiv (1.3)$ Type: straight/mono-connected
5. (1.3 1.2 3.1) / (1.3	3.1 2.1):
Numerical:	$(1.3\ 1.2\ 3.1) \cap (1.3\ 3.1\ 2.1) = (1.3, 3.1)$
Static categorial:	$[\beta \alpha, \alpha, \alpha^{\circ} \beta^{\circ}] \cap [\beta \alpha, \alpha^{\circ} \beta^{\circ}, \alpha^{\circ}] = [\beta \alpha, \alpha^{\circ} \beta^{\circ}]$ Type: straight/mono-connected; detour/mono-connected

Dynamic categorial:	$[[\mathrm{id}1, \beta^{\circ}], [\beta\alpha, \alpha^{\circ}]] \cap [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, \mathrm{id}1]] = [\mathrm{id}1, \beta^{\circ}, \beta\alpha] \equiv (1.1, 3.2, 1.3)$
	Type: intersectional/mono-connected
Ports:	$[\beta\alpha] \cap [\beta\alpha, \alpha^{\circ}\beta^{\circ}] = [\beta\alpha] = (1.3)$
	straight/mono-connected

If straight connections are adjacent, we shall speak of **parallel tracks** (Ch. 3.3.2.). If connections intersect, we call them **joints** or **crossings** (Ch. 3.3.3.). We shall keep the term **detours** (Ch. 3.3.4.) If a track forms a Hamilton circle, we call them **returns** (Ch. 3.3.5.). And finally, if tracks end inside of the network of a semiotic ghost train, we call them **stub tracks** (Ch. 3.3.6.).

According to the law of Trichotomic Triads (cf. Chapter 3.2), all sign classes and reality thematics are connected with the dual-invariant sign class (3.1 2.2 1.3) by at least 1 sub-sign. However, as we have shown in Chapter 2, there is no such law of minimal connectedness amongst dyadic sign connections, since amongst the combinations of transpositions and dual transpositions there are several cases where there are no dyadic connections. Thus, in such places of a semiotic network the semiotic information is interrupted. In order to not let break down the semiotic system which posses several feed-backs due to its symmetries (cf. Toth 2008a) and to not let it end up in a "semiotic catastrophe" (cf. Arin 1983), in such cases one has to change to a dual or non-dual transposition. This possibility, however, can also be chosen when the semiotic information is not interrupted.

Therefore, in the following we shall present some selected trips through semiotic networks, whereby the term "trip" refers to and legitimates itself by the notion of semiosis in dynamic sign connections, implying a movement which can be generative (upgrading) or degenerative (downgrading). Since during his trip a voyager meets many of the semiotic ghosts introduced in Chapter 3.2, I shall call these networks **semiotic ghost trains**, referring to some of my former works (Toth 1998, 2000). As examples, we shall again take the sign class (3.1 2.1 1.3) and some of its transpositions.

3.3.2. Parallel tracks

The following semiotic ghost train, consisting solely of combinations of transpositions with transpositions, shows several both horizontally and vertically parallel tracks:



We will now investigate these parallel tracks according to the classification model presented in Chapter 3.1.

Horizontally parallel tracks:



	3.1) (1.3 3.1 2.1)
[[β, id1], [α°β°, βα]]	[[α°, βα], [βα, α°β°]] [[βα, α°β°], [β°, id1]]
Numerical: Static categorial:	$(2.1 \ 3.1 \ 1.3) \cap (2.1 \ 1.3 \ 3.1) \cap (1.3 \ 3.1 \ 2.1) = (3.1 \ 2.1 \ 1.3)$ $[\alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha] \cap [\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}] \cap [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] = [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$ Type: intersectional/poly-connected
Dynamic categorial:	[[β , id1], [$\alpha^{\circ}\beta^{\circ}$, $\beta\alpha$]] \cap [[α° , $\beta\alpha$], [$\beta\alpha$, $\alpha^{\circ}\beta^{\circ}$]] \cap [[$\beta\alpha$, $\alpha^{\circ}\beta^{\circ}$], [β° , id1]] = [$\alpha^{\circ}\beta^{\circ}$, $\beta\alpha$] = (3.1, 1.3) Type: intersectional/poly-connected
Port:	$[\alpha^{\circ}\beta^{\circ},\beta\alpha] \subset [\alpha^{\circ}\beta^{\circ},\alpha^{\circ},\beta\alpha]$
	2.1) (1.3 2.1 3.1)
$[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]]$	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]]$
Numerical:	$(2.1 \ 1.3 \ 3.1) \cap (1.3 \ 3.1 \ 2.1) \cap (1.3 \ 2.1 \ 3.1) = (3.1 \ 2.1 \ 1.3)$ Type: intersectional/poly-connected
Static categorial: Dynamic categorial:	$[\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}] \cap [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] \cap [\beta\alpha, \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}] = [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$ $[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \cap [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] \cap [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]]$ $= [\alpha^{\circ}\beta^{\circ}] \equiv (3.1)$ Type: intersectional/poly-connected (non-parallel)
Port:	$[\alpha^{\circ}\beta^{\circ}] \subset [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$
	3.1)
[[βα, α°β°], [β°, id1]]	[[α, α°β°], [β, id1]]
Numerical:	$(1.3 \ 3.1 \ 2.1) \cap (1.3 \ 2.1 \ 3.1) = (3.1 \ 2.1 \ 1.3)$ Type: intersectional/poly-connected
Static categorial:	$[\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] \cap [\beta\alpha, \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}] = [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$

Dynamic categorial:	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, \mathrm{id}1]] \cap [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, \mathrm{id}1]] = [\alpha^{\circ}\beta^{\circ}, \mathrm{id}1] \equiv (3.1, 1.1)$
	Type: intersectional/poly-connected
Port:	$[\alpha^{\circ}\beta^{\circ}]; [\alpha^{\circ}\beta^{\circ}, id1] \not\subset [\alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha]$

Vertically parallel tracks:

(3.1 2.1 1.3)	$\begin{bmatrix} \beta^{\circ}, id1 \end{bmatrix}, \begin{bmatrix} \alpha^{\circ}, \beta \alpha \end{bmatrix}$
(2.1 3.1 1.3)	$[[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]]$
(2.1 1.3 3.1)	$[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]]$
(1.3 3.1 2.1)	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]]$
(1.3 2.1 3.1)	[[α , α ° β °], [β , id1]]
Numerical:	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Static categorial:	$\begin{bmatrix} \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}, \beta\alpha \end{bmatrix} \begin{bmatrix} \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha \end{bmatrix} = \begin{bmatrix} \beta\alpha \end{bmatrix}$ $\begin{bmatrix} \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha \end{bmatrix} \begin{bmatrix} \alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ} \end{bmatrix} = \begin{bmatrix} \alpha^{\circ} \end{bmatrix}$ $\begin{bmatrix} \alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ} \end{bmatrix} \begin{bmatrix} \beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ} \end{bmatrix} = \begin{bmatrix} \beta\alpha \end{bmatrix}, \begin{bmatrix} \alpha^{\circ}\beta^{\circ} \end{bmatrix}$ $\begin{bmatrix} \beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ} \end{bmatrix} \begin{bmatrix} \beta\alpha, \alpha^{\circ}, \alpha^{\circ}\beta^{\circ} \end{bmatrix} = \begin{bmatrix} \beta\alpha \end{bmatrix}$
Dynamic categorial:	$ \begin{array}{c} [[\beta^{\circ}, \mathrm{id}1], [\alpha^{\circ}, \beta\alpha]] \parallel [[\beta, \mathrm{id}1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] = [\mathrm{id}1], [\beta\alpha] \\ [[\beta, \mathrm{id}1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] \parallel [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] = \varnothing \\ [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \parallel [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, \mathrm{id}1]] = [\beta\alpha], [\alpha^{\circ}\beta^{\circ}] \\ [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, \mathrm{id}1]] \parallel [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, \mathrm{id}1]] = [\alpha^{\circ}\beta^{\circ}], [\mathrm{id}1] \end{array} $
(3.1 1.3 2.1)	$[[\alpha^{\circ}\beta^{\circ},\beta\alpha],[\alpha,\alpha^{\circ}\beta^{\circ}]]$
(2.1 1.3 3.1)	$[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]]$
(1.3 3.1 2.1)	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]]$
(1.3 2.1 3.1)	$[[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]]$
Numerical:	$\begin{array}{c c} (3.1 \ 1.3 \ 2.1) \\ (2.1 \ 1.3 \ 3.1) = \emptyset \\ (1.3 \ 3.1 \ 2.1) = (1.3), (3.1) \end{array}$
Static categorial:	$ [\alpha^{\circ}\beta^{\circ}, \beta\alpha, \alpha^{\circ}] \parallel [\alpha^{\circ}, \beta\alpha, \alpha^{\circ}\beta^{\circ}] = \emptyset $ $ [\alpha^{\circ}\beta, \beta\alpha, \alpha^{\circ}\beta^{\circ}] \parallel [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] = [\beta\alpha], [\alpha^{\circ}\beta^{\circ}] $ $ [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] \parallel [\beta\alpha, \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}] = \emptyset $
Dynamic categorial:	$[[\alpha^{\circ}\beta^{\circ},\beta\alpha],[\alpha,\alpha^{\circ}\beta^{\circ}]] \parallel [[\alpha^{\circ},\beta\alpha],[\beta\alpha,\alpha^{\circ}\beta^{\circ}]] = [\beta\alpha],[\alpha^{\circ}\beta^{\circ}]$

	$ \begin{bmatrix} [\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}] \end{bmatrix} \ \begin{bmatrix} [\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1] \end{bmatrix} = [\beta\alpha, \alpha^{\circ}\beta^{\circ}] \\ \begin{bmatrix} [\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1] \end{bmatrix} \ \begin{bmatrix} [\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1] \end{bmatrix} = [\alpha^{\circ}\beta^{\circ}], [id1] $
(2.1 3.1 1.3)	$[[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]]$
(1.3 3.1 2.1)	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]]$
(1.3 2.1 3.1)	$[[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]]$
Numerical:	$\begin{array}{c ccccccccccccccccccccccccccccccccccc$
Static categorial:	$[\alpha^{\circ}, \alpha^{\circ}\beta^{\circ}, \beta\alpha] \parallel [\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] = \emptyset$
	$[\beta\alpha, \alpha^{\circ}\beta^{\circ}, \alpha^{\circ}] \parallel [\beta\alpha, \alpha^{\circ}, \alpha^{\circ}\beta^{\circ}] = \emptyset$
Dynamic categorial:	$[[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] \parallel_{\mathbb{R}} [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] = \emptyset$
	$[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, \mathrm{id}1]] \parallel [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, \mathrm{id}1]] = [\alpha^{\circ}\beta^{\circ}], [\mathrm{id}1]$

It has to be pointed out that no path in this network has priority over any other paths, since what a semiotic object is and what semiotic ghosts are is decided exclusively by the variable standpoint of the observer, i.e. by the one who travels through a ghost train.



http://www.buschtunnel.de/schaeden/images/sim_buschtunnel_20061129_11b.jpg

3.3.3. Joints and crossings

In the semiotic network given in Chapter 3.3.2, we will now let away the parallel tracks that we had also scrutinized in the last chapter and concentrate on the crossings of connecting tracks that serve as joints in the trip through the semiotic ghost train:



We find the following types of crossings. In order to research the respective tracks, we use the transitional classes introduced in Chapter 1.4):

 $(3.1 \ 2.1) \rightarrow (2.1 \ 3.1) \Leftrightarrow [\beta^{\circ}, \operatorname{id1}] \rightarrow [\beta, \operatorname{id1}]$ Transitional class: $[\operatorname{id1}] \equiv (1.1)$ $(2.1 \ 1.3) \rightarrow (1.3 \ 2.1) \Leftrightarrow [\alpha^{\circ}, \beta\alpha] \rightarrow [\alpha, \alpha^{\circ}\beta^{\circ}]$ Transitional class: \emptyset $(3.1 \ 1.3) \rightarrow (1.3 \ 3.1) \Leftrightarrow [\alpha^{\circ}\beta^{\circ}, \beta\alpha] \rightarrow [\beta\alpha, \alpha^{\circ}\beta^{\circ}]$ Transitional class: $[\beta\alpha] \equiv (1.3)$ $(2.1 \ 1.3 \ 3.1) \Rightarrow (1.3 \ 3.1 \ 2.1) \Leftrightarrow [[\alpha^{\circ}, \beta\alpha] \ [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \rightarrow [[\beta\alpha, \alpha^{\circ}\beta^{\circ}]]$

 $(2.1 \ 1.3 \ 3.1) \rightarrow (1.3 \ 3.1 \ 2.1) \Leftrightarrow [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \rightarrow [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]]$ Transitional class: $[\beta\alpha, \alpha^{\circ}\beta^{\circ}] \equiv (1.3, 3.1)$

An amazing result is that the two dyadic sign-relations (2.1 1.3) and (1.3 2.1) have no transition class although their paths cross one another. A closer look at other mirror-inverted pairs of dyadic relations shows that the respective transition class is always then empty when a dyadic relation does not contain identical sub-signs (1.1, 2.2, 3.3) or has not the structure (X, Y°), (Y°, X). In the following, we list all possible 36 combinations; the ones with empty transition class are in bold. Further, the categorial structures (X, X°), (X°, X) which are the only pairs of dyadic relations with transitional classes containing two elements are underlined:

(1.1) (1.2) (1.2, 1.1)	[id1, α] [id1, α°]
(1.1) (1.3) (1.3, 1.1)	[id1, βα] [id1, α°β°]
(1.1) (2.1) (2.1, 1.1)	[α, id1] [α°, id1]
(1.1) (2.2) (2.2, 1.1)	[α, α] [α°, α°]
(1.1) (2.3) (2.3, 1.1)	[α, βα] [α°, α°β°]
(1.1) (3.1) (3.1, 1.1)	$[\beta \alpha, id1] [\alpha^{\circ}\beta^{\circ}, id1]$
(1.1) (3.2) (3.2, 1.1)	[βα, α] [α°β°, α°]
(1.1) (3.3) (3.3, 1.1)	$[\beta \alpha, \beta \alpha] [\alpha^{\circ} \beta^{\circ}, \alpha^{\circ} \beta^{\circ}]$
/· ·· ·· /· ·· ·· ·· ··	
(1.2) (1.3) (1.3) (1.2)	[id1, β] [id1, β°]
(1.2) (2.1) (2.1) (1.2)	$[\alpha, \alpha^{\circ}] [\alpha^{\circ}, \alpha]$
(1.2) (2.2) (2.2) (1.2)	[α, id2], [α°, id2]
(1.2) (2.3) (2.3) (1.2)	[α, β], [α°, β°]
(1.2) (3.1) (3.1) (1.2)	[βα, α°], [α°β°, α]
(1.2) (3.2) (3.2) (1.2)	[βα, id2], [α°β°, id2]
(1.2) (3.3) (3.3) (1.2)	[βα, β], [α°β°, β°]
(1.3) (2.1) (2.1) (1.3)	[α, α°β°], [α°, βα]
(1.3) (2.2) (2.2) (1.3)	[α, β°], [α°, β]
(1.3) (2.3) (2.3) (1.3)	$[\alpha, id3], [\alpha^{\circ}, id3]$



 $(1.3) (3.1) (3.1) (1.3) [\beta \alpha, \alpha^{\circ} \beta^{\circ}], [\alpha^{\circ} \beta^{\circ}, \beta \alpha]$ (1.3) (3.2) (3.2) (1.3) [\beta \alpha, \beta^{\circ}], [\alpha^{\circ} \beta^{\circ}, \beta]

(2.1) (3.2) (3.2) (2.1) $[\beta, \alpha], [\beta^{\circ}, \alpha^{\circ}]$

 $[\beta\alpha, id3], [\alpha^{\circ}\beta^{\circ}, id3]$

[id2, $\beta\alpha$], [id2, $\alpha^{\circ}\beta^{\circ}$] [β , id1], [β° , id1]

 $[\beta, \beta\alpha], [\beta^{\circ}, \alpha^{\circ}\beta^{\circ}]$

 $[id2, \alpha], [id2, \alpha^{\circ}]$

(1.3) (3.3) (3.3) (1.3)

(2.1) (2.2) (2.2) (2.1)

(2.1) (2.3) (2.3) (2.1)

(2.1) (3.1) (3.1) (2.1)

(2.1) (3.3) (3.3) (2.1)

Quelle: http://www.laufenburg.de/bilder/news/big2/gleise.jpg

3.3.4. Detours

As starting base we shall take again the small network given in Chapter 3.3.2, highlighting now a thorough path with some detours:



The static categorial structure through this ghost train is as follows:

In a static view, there is no connection between (2.1 1.3 3.1) and (1.3 3.1 2.1), since the transition class between these two transpositions is empty. Now let us have a look at the dynamic categorial structure:

 $[[\beta^{\circ}, id1], [\alpha^{\circ}, \beta\alpha]] \parallel [[\alpha^{\circ}\beta^{\circ}, \beta\alpha], [\alpha, \alpha^{\circ}\beta^{\circ}]] = [\beta\alpha]$ $[[\alpha^{\circ}\beta^{\circ}, \beta\alpha], [\alpha, \alpha^{\circ}\beta^{\circ}]] \cap [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha]$ $[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \parallel [[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] = [\beta\alpha, \alpha^{\circ}\beta^{\circ}]$ $[[\beta, id1], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] \cap [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha]$ $[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] \parallel [[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] = [\beta\alpha, \alpha^{\circ}\beta^{\circ}]$ $[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] \cap [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] = [\alpha^{\circ}\beta^{\circ}, id1]$ $[[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] \cap [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] = [\alpha^{\circ}\beta^{\circ}]$ $[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]] \cap [[\alpha, \alpha^{\circ}\beta^{\circ}], [\beta, id1]] = [\alpha^{\circ}\beta^{\circ}, id1]$

Thus, we have as transitional class $[\beta\alpha] \Leftrightarrow (1.3)$ from $[\beta\alpha]$ to $[\beta\alpha, \alpha^{\circ}\beta^{\circ}]$, when again a thourough track seems to stop, but if we calculate the transitional class between $[\beta\alpha, \alpha^{\circ}\beta^{\circ}]$

 \Leftrightarrow (1.3, 3.1) and $[\alpha^{\circ}\beta^{\circ}, id1] \Leftrightarrow$ (3.1, 1.1), we get (3.1) \Leftrightarrow $[\alpha^{\circ}\beta^{\circ}]$, thus $[\alpha^{\circ}\beta^{\circ}]$ establishing the "bridge" between the transitional classes $[\beta\alpha]$ and $[\alpha^{\circ}\beta^{\circ}]$. If we now return to the static structures (2.1 1.3 3.1) and (1.3 3.1 2.1) and turn these transpositions into dynamic categorial structure, we get (2.1 1.3 3.1) \Leftrightarrow $[[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]]$ and (1.3 3.1 2.1) \Leftrightarrow $[[\beta\alpha, \alpha^{\circ}\beta^{\circ}], [\beta^{\circ}, id1]]$ and hence the transitional class (3.1, 1.3) \Leftrightarrow $[\alpha^{\circ}\beta^{\circ}, \beta\alpha]$. This result is the more interesting because we have:

$$\begin{split} & [\alpha^{\circ}\beta^{\circ}], \, [\beta\alpha] \subset [\alpha^{\circ}\beta^{\circ}, \, \beta\alpha] \\ & \times [\beta\alpha] = [\alpha^{\circ}\beta^{\circ}] \end{split}$$

In other words: In the static view, the transitional class between (2.1 1.3 3.1) and (1.3 3.1 2.1) is empty, because a direct transition from $(1.3) \rightarrow (3.1)$ cannot be taken into account, while in a dynamic view, where the mutual relations of triadic as well as trichotomic place-values of the sub-signs are taken into account, dual relations like $(1.3) \times (3.1)$ go into the categorial structure and thus enable transitional classes between transpositions whose paths are intersecting one another.

3.3.5. Returns

Up to now, we have restricted ourselves to trips from the left to the right and from top to bottom. Let us now have look to trips that lead back to the starting point, using a part of the network presented in Chapter 3.3.4:



Static categorial structure:

Dynamic categorial structure:

 $\begin{array}{c} [[\beta^{\circ}, \operatorname{id1}], [\alpha^{\circ}, \beta\alpha]] & \| [[\alpha^{\circ}\beta^{\circ}, \beta\alpha], [\alpha, \alpha^{\circ}\beta^{\circ}]] = [\beta\alpha] \\ [[\alpha^{\circ}\beta^{\circ}, \beta\alpha], [\alpha, \alpha^{\circ}\beta^{\circ}]] \cap [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] = [\alpha^{\circ}\beta^{\circ}, \beta\alpha] \\ [[\alpha^{\circ}, \beta\alpha], [\beta\alpha, \alpha^{\circ}\beta^{\circ}]] & \| [[\beta, \operatorname{id1}], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] = [\beta\alpha, \alpha^{\circ}\beta^{\circ}] \\ [[\beta, \operatorname{id1}], [\alpha^{\circ}\beta^{\circ}, \beta\alpha]] \cap [[\beta^{\circ}, \operatorname{id1}], [\alpha^{\circ}, \beta\alpha]] = [\operatorname{id1}, \beta\alpha] \end{array}$

While the static categorial structure of this return-path does not show a common morphism, the dynamic categorial structure shows that by using the "track" $[\beta\alpha] \Leftrightarrow (1.3)$ and visiting the ghosts represented by the above transpositions, one gets back to the starting point in this semiotic ghost train:



If there is no transition class and thus no common morphism between two transpositions, then the information transfer in a semiotic network comes to stop. The basic two different types of stub tracks are presented in the next chapter.

3.3.6. Stub tracks

If we have again a look at our semiotic network given in Chapter 3.3.3, we recognize that the paths at the rightmost transpositions are only connected with the ones to their left, but not downwards:



Thus, the semiotic ghosts (2.1 1.3 3.1), (1.3 2.1 3.1) and (1.3 2.1 3.1) in these positions of the semiotic ghost train mark stub tracks, unless somebody manages to make a detour (Chapter 3.3.4.) to the left. Therefore, at these nodes of the network the semiotic information is not transferred downwards. In order to describe this process, we refer to the notion of "semiotic catastrophe", introduced by Arin (1981, pp. 360 ss.). Arin analyzes the semiotic catastrophe of the sign class (3.2 2.2 1.2) and its reality thematic (2.1 2.2 2.3) as follows:



Applying Arin's method of analyzing sign relations into three pairs of dyadic relations, then into their constitutive sub-signs and finally into their monadic categories according to triadic order in the case of sign classes and trichotomic order in the case of reality thematics, we get the following scheme of semiotic catastrophe for the three above transpositions serving as semiotic stub tracks:



whereby the last two semiotic catastrophes are identical by their numerical and categorial schemes, but not by their position in the network.



http://www.rodgau-bahn.de/Dietzenbach/Steinberg/Industriegleis/P7260467.JPG
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